Math 102, Fall 2022 — Homework 10

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Due December 9 at 5:00 pm

Instructions. This problem set has material from Week 13 of class.

Problem 1. Approximate the given function value with the indicated Taylor polynomial and then use Taylor's theorem to estimate the error of the approximation.

a. Approximate $\cos 1$ with the Maclaurin polynomial of degree 4.

b. Approximate $\ln 1.5$ with the Taylor polynomial of $\ln x$ of degree 3 centered at $x = 1$.

c. Approximate $\sqrt[3]{9}$ with the Taylor polynomial of degree 2 centered at $x = 8$.

Problem 2. Find n so that the Taylor polynomial $p_n(x)$ centered at c and evaluated at a approximates $f(a)$ to within ϵ of the actual value.

a. $f(x) = e^x, c = 0, a = 2, \epsilon = 0.0001$

b. $f(x) = \cos x, c = 0, a = 2\pi/3, \epsilon = 0.0001$

Problem 3. Find the first 4 non-zero terms of the Maclaurin series for the following functions using known Maclaurin series.

a. e^{-x^2}

b. $x \sin x$

c. $\ln(1 - x)$

d. $\arctan x^2$

Problem 4. Find the sums of the following series.

a. $\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$

b. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

c. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!}$

d. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots$