

## Math 102, Fall 2022 — Homework 2

Tim Chumley

Due September 23 at 5:00 pm

**Instructions.** This problem set has material mostly from Week 2 of class.

**Problem 1.** Answer the following questions with a short justification.

- True or False: if  $\{a_n\}$  is a convergent sequence, then  $\sum_{n=1}^{\infty} a_n$  is a convergent series.
- True or False: if  $\{a_n\}$  converges to 0, then  $\sum_{n=1}^{\infty} a_n$  converges.
- Suppose that  $\sum_{n=1}^{\infty} a_n$  is a convergent series and  $\{b_n\}$  and  $\{c_n\}$  are sequences such that  $0 \leq b_n \leq a_n \leq c_n$  for all integers  $n \geq 1$ . What can be concluded (in relation to convergence) about the series  $\sum_{n=1}^{\infty} b_n$  and  $\sum_{n=1}^{\infty} c_n$ ?

**Problem 2.** For each of the following series  $\sum_{n=1}^{\infty} a_n$  do the following: (1) compute  $\lim_{n \rightarrow \infty} a_n$  and (2) explain what you can conclude about the series from the  $n$ th term test for divergence.

- $\sum_{n=1}^{\infty} \frac{n^3 - 2n^2}{5n^3 + 1}$
- $\sum_{n=1}^{\infty} \frac{n+1}{2n+3}$
- $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

**Problem 3.** For each of the following series  $\sum_{n=1}^{\infty} a_n$ , do the following: (1) decide on a series  $\sum_{n=1}^{\infty} b_n$  to compare it to, (2) make a conjecture about whether  $\sum_{n=1}^{\infty} a_n$  converges or diverges, and (3) justify your conjecture using the Comparison Test.

- $\sum_{n=1}^{\infty} \frac{n+1}{3n^2-2}$
- $\sum_{n=1}^{\infty} \frac{3^n - n}{4^n}$
- $\sum_{n=1}^{\infty} \frac{2^{-n}}{n}$

**Problem 4.** For each of the following series  $\sum_{n=1}^{\infty} a_n$ , do the following: (1) decide on a series  $\sum_{n=1}^{\infty} b_n$  to compare it to, (2) make a conjecture about whether  $\sum_{n=1}^{\infty} a_n$  converges or diverges, and (3) justify your conjecture using the Limit Comparison Test.

a.  $\sum_{n=1}^{\infty} \frac{n+1}{n^3+6}$

b.  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2(n+2)}}$

c.  $\sum_{n=1}^{\infty} \frac{n}{n+n^2}$