

# Math 102, Fall 2022 — Homework 3

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Due September 30 at 5:00 pm

**Instructions.** This problem set has material mostly from Week 3 of class.

**Problem 1.** For each of the following series, use the Ratio Test to decide whether it converges or diverges.

a.  $\sum_{n=1}^{\infty} \frac{n^4}{5^n}$

b.  $\sum_{n=1}^{\infty} \frac{1}{ne^n}$

c.  $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

**Problem 2.** For each of the following series  $\sum_{n=1}^{\infty} a_n$ , do two things: (1) explain why it fails to be absolutely convergent (that is, explain why  $\sum_{n=1}^{\infty} |a_n|$  diverges using an appropriate test), and (2) explain why it is conditionally convergent using the alternating series test.

a.  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

b.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{4n+3}}$

c.  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n^{1/3}+1}$

**Problem 3.** For each of the following series, say whether the series is absolutely convergent, conditionally convergent, or divergent and explain why using appropriate tests.

a.  $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$

b.  $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

c.  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n^2(n+2)}}$

d.  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$

**Problem 4.** For each part below, give an example of a series  $\sum_{n=1}^{\infty} a_n$  with the given condition or explain why it is not possible.

a.  $\sum_{n=1}^{\infty} a_n$  converges but  $\sum_{n=1}^{\infty} |a_n|$  diverges.

b.  $\sum_{n=1}^{\infty} |a_n|$  converges but  $\sum_{n=1}^{\infty} a_n$  diverges.