Math 102, Fall 2022 — Homework 4

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Due October 14 at 5:00 pm

Instructions. This problem set has material mostly from Week 4 of class.

Problem 1. Find the following indefinite integrals by first doing some simplifying algebra.

a.
$$\int x^3(x^2+1) dx$$

b.
$$\int \left(\sqrt{x^3} - \frac{2}{x}\right) dx$$

c.
$$\int \left(\frac{x+1}{x}\right) dx$$

Problem 2. Find the general antiderivative of each of the following functions. Note this is asking for the same thing as the previous problem, just with different terminology.

a.
$$f(x) = 4x - \frac{1}{\sqrt{x}} + 9x^{3.5}$$

b.
$$f(x) = 3x^3 - 4e^x + 8$$

c.
$$f(x) = 7\sin x - 4\cos x$$

Problem 3. Find the following definite integrals. To make things easier for the graders, give both an exact final answer like $\ln 3 + e^3$ and a decimal final answer (to 4 decimal places) like $\ln 3 + e^3 \approx 21.1842$.

a.
$$\int_0^3 (x^2 + 3x^{0.2} - 4) dx$$

b.
$$\int_{0}^{2} 3e^{x} dx$$

c.
$$\int_0^1 (\sin \theta + \cos \theta) \, d\theta$$

Problem 4. Find the following areas using definite integrals.

- a. The area between the curves $y = 5e^x$, y = x, x = 0, and x = 3.
- b. The bounded area between the curves $y = x^2$ and $y = 2 x^2$.
- c. The bounded region between the x-axis and the graph of $y = x^3 x$.

Problem 5. Suppose

$$\int_0^2 f(x) \, dx = 5, \int_0^3 f(x) \, dx = 7, \int_0^2 g(x) \, dx = -3, \int_0^3 g(x) \, dx = 5.$$

Using this information and algebraic properties of the integral, compute the following definite integrals.

a.
$$\int_0^2 [f(x) + g(x)] dx$$

b.
$$\int_0^3 [f(x) - g(x)] dx$$

c.
$$\int_2^3 [3f(x) + 2g(x)] dx$$