Math 102, Fall 2021 — Homework 4

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Due September 29 at 5:00 pm

Instructions. This homework, like most others this semester, has two parts. One part is on Webwork, and the other part is some problems that you will write solutions to by hand and submit on Gradescope.

Webwork

See Webwork for a set of problems from Section 9.4.

Written problems

Write up solutions to the following problems, making sure to show your work, write neatly, scan clearly, and generally follow the guidelines for writing good homework solutions. You should submit solutions on Gradescope.

Problem 1. For each of the following series, use the Ratio Test to decide whether it converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{n^4}{5^n}$ 2. $\sum_{n=1}^{\infty} \frac{1}{ne^n}$ 3. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

Problem 2. For each of the following series, first state what conclusion can be drawn from the alternating series test. Second, say whether the series is absolutely convergent, conditionally convergent, or divergent and explain why.

1. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$ 2. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$ 3. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{e^n}$

Problem 3. For each of the following series, say whether the series is absolutely convergent, conditionally convergent, or divergent and explain why. In your explanations

- If you believe the series is absolutely convergent, explain why $\sum |a_n|$ converges (eg. use the ratio test, or compare to a convergent *p*-series).
- If you believe the series is conditionally convergent explain why $\sum |a_n|$ diverges (eg. compare with a divergent *p*-series) and explain why $\sum a_n$ converges (eg. use the alternating series test).
- If you believe the series diverges, explain why (eg. use the ratio test or *n*th term test).

1.
$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$$

2.
$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$$

- 3. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4+7}$
- 4. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n^2(n+2)}}$
- 5. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$ 6. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$