Math 102, Fall 2022 — Homework 8

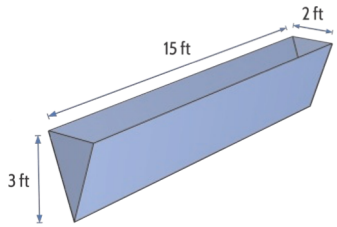
Tim Chumley

Due November 18 at 5:00 pm

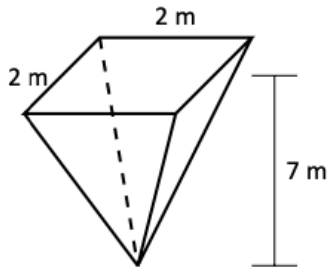
Instructions. This problem set has material from Week 9 and Week 10 of class.

Problem 1. A force of 7 Newtons is required to compress a spring with natural length 21 meters to a length of 11 meters. How much work is performed in compressing it from a length of 15 meters to a length of 12 meters?

Problem 2. The tank shown below (whose dimensions we'll assume are in meters, not feet) is filled with a fluid that has a density of 60 kg per cubic meter. The water is to be pumped to a point 2 meters above the top of the tank. How much work is performed in pumping all the fluid from the tank? Set up but do not compute an integral for this quantity. Please use Wolfram Alpha or another piece of software to compute the value of the integral.



Problem 3. A water tank in the shape of an inverted pyramid with dimensions in the figure below (height 7 meters, square base with 2 meter side lengths) is filled with water, which has a density of 1000 kg per cubic meter. It is filled to a height of 4 meters. Find the work performed in pumping the water to a point 5 meters above the top of the tank. Set up but do not compute an integral for this quantity. Please use Wolfram Alpha or another piece of software to compute the value of the integral.



Problem 4. A conical water tank is 5 meters tall and has a top radius of 3 meters. The tank is filled with water, which has density of 1000 kg per cubic meter. Please use Wolfram Alpha or another piece of software to compute the value of the integrals in terms of π.

a. Find the work performed in pumping all the water to the top of the tank.

b. Find the work performed in pumping the top 2.5 meters of water to the top of the tank.

c. Find the total volume of the tank.

Problem 5. Use the integral test to explain whether the following series converge.

a. $\sum_{n=1}^{\infty} \frac{1}{n^5}$

b. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

c. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

d. $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$