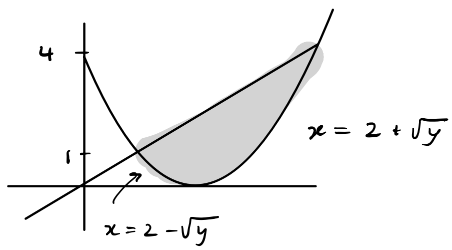
Problem 1. Sketch the region enclosed by the given curves and then set up integrals for the area of the region in two ways: with respect to x and with respect to y.

a. $y = (x-2)^2, \quad y = x$

b. $y = (x-1)^2 - 1, \quad y = x + 4$

a)



Intersection points:

$$x = (x-2)^2$$

$$x = x^2 - 4x + 4$$

$$0 = x^2 - 5x + 4$$

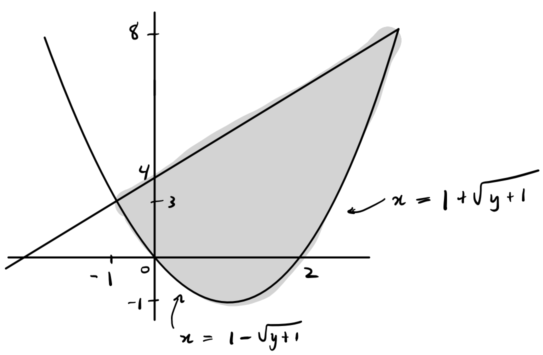
$$= (x-4)(x-1)$$

$$x = 1, 4$$

$$\int_1^4 \left(x - (x-2)^2\right) dx$$

$$= \int_0^1 \left(2+\sqrt{y} - (2-\sqrt{y})\right) dy + \int_1^4 \left(2+\sqrt{y} - y\right) dy$$

b)



Intersection points

$$x^2 - 2x = x + 4$$

$$\Leftrightarrow \quad x^2 - 3x - 4 = 0$$

$$\Leftrightarrow \quad (x-4)(x+1) = 0$$

$$x = -1, 4$$

$$\text{Area} = \int_{-1}^{4} \left[(x+4) - (x^2 - 2x)\right] dx$$

$$= \int_{-1}^{3} \left((1+\sqrt{y+1}) - (1-\sqrt{y+1})\right) dy + \int_3^8 \left(1+\sqrt{y+1} - (y-4)\right) dy$$

Problem 2. Consider the regions R_1, R_2, R_3 below. For each region, set up an integral for its area in two ways: with respect to x and with respect to y.

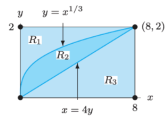


Figure 8.106

$$\text{area}(R_1) = \int_0^8 (2 - x^{1/3})\,dx = \int_0^2 y^3\,dy$$

$$\text{area}(R_2) = \int_0^8 \left(x^{1/3} - \tfrac{1}{4}x\right)dx = \int_0^2 (4y - y^3)\,dy$$

$$\text{area}(R_3) = \int_0^8 \tfrac{1}{4}x\,dx = \int_0^2 (8 - 4y)\,dy$$

Problem 3. Consider the solid shown below. Set up an integral for its volume, given that the radius r of the circular slice at h is $\sqrt{h}$.

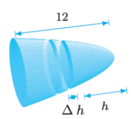
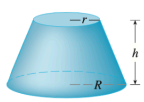
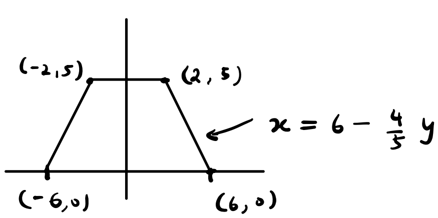


Figure 8.107

$$\text{volume} = \int_0^{12} \pi(\sqrt{h})^2\,dh$$

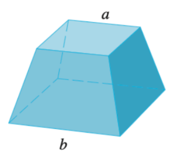
Problem 4. Consider the solid shown below. Set up an integral for its volume, given that $r = 2, R = 6$ and $h = 5$.

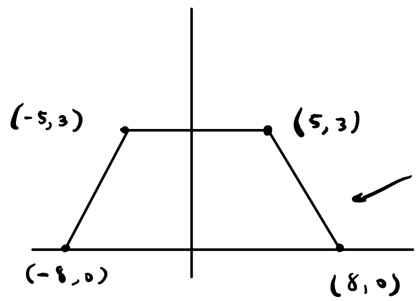




$$\text{volume} = \int_0^5 \pi\left(6 - \tfrac{4}{5}y\right)^2 dy$$

Problem 5. Consider the solid shown below whose base is a square with side length b and whose top is a square with side length a. Set up an integral for its volume, given that $a = 10$ and $b = 16$ and its height is 3.





$x = 8 - y$

$$\text{volume} = \int_0^3 (8-y)^2 \, dy$$

Problem 6. Suppose that f and g are functions that have a vertical asymptote at $x = 0$ but are continuous for all $x > 0$. Further, suppose that $0 \le g(x) \le f(x)$ for all $x > 1$. Please answer the following true false questions.

a. If $\int_1^\infty f(x)\,dx$ converges then $\int_1^\infty g(x)\,dx$ converges.

b. If $\int_1^\infty f(x)\,dx$ diverges then $\int_1^\infty g(x)\,dx$ diverges.

c. If $\int_0^1 f(x)\,dx$ diverges but $\int_1^\infty f(x)\,dx$ converges, then $\int_0^\infty f(x)\,dx$ converges.

(a) true

(b) false : example: $\frac{1}{x^2} < \frac{1}{x}$ for all $x > 1$

and $\int_1^\infty \frac{1}{x}\,dx$ diverges but $\int_1^\infty \frac{1}{x^2}\,dx$ converges

(c) false : in order for $\int_0^\infty f(x)dx$ to converge

we need <u>both</u> $\int_0^1 f(x)dx$ and $\int_1^\infty f(x)dx$ to converge.