§ 9.1 Sequences

Def A <u>sequence</u> is an infinite, ordered list of numbers:

$$(s_n) = (s_1, s_2, s_3, s_4, \dots)$$

Examples List the first 5 terms of the following sequences:

Unless otherwise stated assume we start with $n=1$.

(a) $s_n = \dfrac{n(n+1)}{2}$

$$\underbrace{\frac{1\cdot 2}{2}}_{n=1}, \underbrace{\frac{2\cdot 3}{2}}_{n=2}, \underbrace{\frac{3\cdot 4}{2}}_{n=3}, \frac{4\cdot 5}{2}, \frac{5\cdot 6}{2} = 1, 3, 6, 10, 15$$

$\dots$

(b) $s_n = \dfrac{n+(-1)^n}{n}$

$$\frac{1+(-1)^1}{1}, \frac{2+(-1)^2}{2}, \frac{3+(-1)^3}{3}, \frac{4+(-1)^4}{4}, \frac{5+(-1)^5}{5}$$

$$= 0, \frac{3}{2}, \frac{2}{3}, \frac{5}{4}, \frac{4}{5}$$

(c) $s_n = \dfrac{2n-1}{n^2}$ $\qquad 1, \dfrac{3}{4}, \dfrac{5}{9}, \dfrac{7}{16}, \dfrac{9}{25}$

Example Give a formula for the general term s_n of the following sequences:

(a) $2, 4, 6, 8, 10, 12, \ldots$ $\quad s_n = 2n$

(b) $2, -4, 8, -16, 32, -64, \ldots$ $\quad s_n = (-1)^{n+1} 2^n$

(c) $\frac{1}{7}, \frac{3}{7}, \frac{9}{7}, \frac{27}{7}, \frac{81}{7}, \ldots$ $\quad s_n = \frac{3^{n-1}}{7}$

Def We say a sequence (s_n) converges if there is a number L that the terms of (s_n) get arbitrarily close to for sufficiently large n. We call L the limit of the sequence ($L = \lim_{n\to\infty} s_n$) and say that (s_n) converges to L. If no such L exists, we say (s_n) diverges.

Example Which of the following sequences converge? For those that converge, what is the limit?

(a) $s_n = (0.8)^n$ converges to 0

(b) $s_n = (1.7)^n$ diverges

(c) $s_n = (-0.4)^n$ converges to 0

(d) $s_n = (-1)^n$ diverges

(e) $s_n = (-2.3)^n$ diverges

Fact Let $x \in \mathbb{R}$. Then the sequence $s_n = x^n$ converges when $|x| < 1$ or $x = 1$ and diverges when $|x| > 1$ or $x = -1$.

Example Compute $\lim_{n\to\infty} s_n$ or show (s_n) diverges

(a) $s_n = \frac{1}{n} + (-1)^n$ diverges since $(-1)^n$ diverges

(b) $s_n = \frac{1+(-1)^n}{n}$

$$0 \le \frac{1+(-1)^n}{n} \le \frac{2}{n}$$

Since $\lim_{n\to\infty} 0 = 0$ and $\lim_{n\to\infty} \frac{2}{n} = 0$, $\lim_{n\to\infty} \frac{1+(-1)^n}{n} = 0$.

Squeeze Theorem If $a_n \le b_n \le c_n$ for all n and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$.