(b) $S_n = \dfrac{1-e^{-n}}{1+e^{-n}}$ $\quad \lim\limits_{n\to\infty} \dfrac{1-e^{-n}}{1+e^{-n}} = \dfrac{1-\lim\limits_{n\to\infty} e^{-n}}{1+\lim\limits_{n\to\infty} e^{-n}} = 1$

(c) $S_n = \dfrac{3n+5}{7n^2+9n+11}$

$$\lim_{n\to\infty} \frac{3n+5}{7n^2+9n+11} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{\frac{3}{n}+\frac{5}{n^2}}{7+\frac{9}{n}+\frac{11}{n^2}} = \frac{0}{7} = 0.$$

(d) $S_n = \dfrac{2n^4+6}{8n^3+17}$

$$\lim_{n\to\infty} \frac{2n^4+6}{8n^3+17} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{2+\frac{6}{n^4}}{\frac{8}{n}+\frac{17}{n^4}} = \infty$$

(e) $S_n = \dfrac{3n^5+4n^2+2}{5n^5+3n^3}$

$$\lim_{n\to\infty} \frac{3n^5+4n^2+2}{5n^5+3n^3} \cdot \frac{\frac{1}{n^5}}{\frac{1}{n^5}} = \lim_{n\to\infty} \frac{3+\frac{4}{n}+\frac{2}{n^5}}{5+\frac{3}{n^2}} = \frac{3}{5}$$

Fact Suppose P and Q are polynomials.

(1) if $\deg P < \deg Q$, then $\lim\limits_{n\to\infty} \dfrac{P(n)}{Q(n)} = 0$

(2) if $\deg P > \deg Q$, then $\lim\limits_{n\to\infty} \dfrac{P(n)}{Q(n)} = \pm\infty$

(3) if $\deg P = \deg Q$, then $\lim\limits_{n\to\infty} \dfrac{P(n)}{Q(n)}$ is the ratio of the leading coefficients.

Problem 1. Write out the first 5 terms of each of the following sequences, given the formula s_n where $n \geq 1$.

a. $s_n = 2^n + 1$

b. $s_n = \frac{2n}{2n+1}$

c. $s_n = (-1)^{n+1} \left(\frac{1}{2}\right)^{n-1}$

(a) $3, 5, 9, 17, 33$

(b) $\frac{2}{3}, \frac{4}{5}, \frac{6}{7}, \frac{8}{9}, \frac{10}{11}$

(c) $1, \frac{-1}{2}, \frac{1}{4}, \frac{-1}{8}, \frac{1}{16}$

Problem 2. Given the first few terms of the sequence, find a general formula for s_n where $n \geq 1$.

a. $4, 8, 16, 32, 64, \ldots$

b. $1/3, 2/5, 3/7, 4/9, 5/11, \ldots$

c. $1/2, -1/4, 1/6, -1/8, 1/10, \ldots$

(a) 2^{n+1}

(b) $\frac{n}{2n+1}$

(c) $\frac{(-1)^{n+1}}{2n}$

Problem 3. For each of the following sequences, find its limit or state that the sequence diverges. You might find it helpful to list out the first few terms of the sequence to get a sense of its behavior as n gets larger.

a. $s_n = (-0.9)^n$

b. $s_n = 2^n$

c. $s_n = 3^{-n}$

d. $s_n = 5 + e^{-3n}$

e. $s_n = \frac{1}{n} + n^2$

(a) $\lim_{n\to\infty} (-0.9)^n = 0$

(b) $\lim_{n\to\infty} 2^n = \infty$ so diverges

(c) $\lim_{n\to\infty} 3^{-n} = \lim_{n\to\infty} \left(\frac{1}{3}\right)^n = 0$

(d) $\lim_{n\to\infty} 5 + e^{-3n} = 5 + \lim_{n\to\infty} \left(\frac{1}{e^3}\right)^n = 5$

(e) $\lim_{n\to\infty} \frac{1}{n} + n^2 = \infty$ so diverges

Problem 4. For each of the following sequences, find its limit or state that the sequence diverges.

a. $s_n = \frac{n+3}{2n^2+1}$

b. $s_n = \frac{2n^2+1}{7n+3}$

c. $s_n = \frac{11n^3+3n^2}{5n^3+7n+1}$

d. $s_n = \frac{\cos(\pi n)}{n^2}$

e. $s_n = \frac{4n+(-1)^n 7}{8n+(-1)^n 9}$

(a) 0 (b) ∞, so diverges

(c) $\frac{11}{5}$ (d) $\frac{-1}{n^2} \le \frac{\cos(\pi n)}{n^2} \le \frac{1}{n^2}$,

So $\lim_{n\to\infty} \frac{\cos(\pi n)}{n^2} = 0$ by squeeze theorem

(e) $\frac{4n-7}{8n+9} \le \frac{4n+(-1)^n 7}{8n+(-1)^n 9} \le \frac{4n+7}{8n-9}$

So $\lim_{n\to\infty} \frac{4n+(-1)^n 7}{8n+(-1)^n 9} = \frac{4}{8} = \frac{1}{2}$ by squeeze theorem

Next time we'll begin discussing infinite series.

A sequence is an ordered list of numbers.

A series is an ordered sum of numbers.

We'll talk about series like

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$

and $$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots$$

and what it means for them to converge or diverge.

What do you think the definition of convergence should be?

Which of the above do you think converge or diverge?