§7.1 Substitution with definite integrals

How do we handle definite integrals?

By changing the limits of integration!

Example

① $\displaystyle\int_0^2 x e^{x^2}\,dx$

$u = x^2$

$\dfrac{du}{dx} = 2x$

$\frac{1}{2}\,du = x\,dx$

$x = 0,\ u = 0$

$x = 2,\ u = 2^2 = 4$

$$= \frac{1}{2}\int_0^4 e^u\,du$$

$$= \frac{1}{2}\,e^u\Big|_0^4$$

$$= \frac{1}{2}\left(e^4 - 1\right)$$

(2) $\displaystyle\int_1^4 \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

$u = \sqrt{x} = x^{1/2}$

$\dfrac{du}{dx} = \dfrac{1}{2}x^{-1/2}$

$2\,du = x^{-1/2}\,dx$

$x=1,\ u=1$

$x=4,\ u=2$

$= \displaystyle\int_1^4 (\cos\sqrt{x})\, x^{-1/2}\,dx$

$= 2\displaystyle\int_1^2 \cos u\,du$

$= 2\sin u\Big|_1^2$

$= 2\left(\sin(2) - \sin(1)\right)$

radians

Challenge problems

(1) $$\int \sqrt{1+\sqrt{x}}\, dx$$

$u = 1+\sqrt{x} \Rightarrow \sqrt{x} = u-1$

$du = \frac{1}{2} x^{-1/2}\, dx$

$dx = 2\sqrt{x}\, du$

$= 2(u-1)du$

$$= \int \sqrt{u} \cdot 2(u-1)\, du$$

$$= 2\int \sqrt{u}\,(u-1)\, du$$

$$= 2\int (u^{3/2} - u^{1/2})\, du = 2\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{4}{5}(1+\sqrt{x})^{5/2} - \frac{4}{3}(1+\sqrt{x})^{3/2} + C$$

(2) $$\int (x+7)\sqrt[3]{3-2x}\, dx$$

$u = 3-2x \Rightarrow x = -\frac{1}{2}(u-3)$

$du = -2dx$

$$= \int \left(-\frac{1}{2}(u-3) + 7\right)\left(\sqrt[3]{u}\right)\left(-\frac{1}{2}du\right)$$

$$= -\frac{1}{2}\int \left(-\frac{1}{2}u + \frac{17}{2}\right) u^{1/3}\, du$$

$$= -\frac{1}{2}\int \left(-\frac{1}{2}u^{4/3} + \frac{17}{2}u^{1/3}\right) du$$

$$= \frac{1}{4}\int (u^{4/3} - 17u^{1/3})\, du$$

$$= \frac{1}{4}\left(\frac{3}{7}u^{7/3} - 17\left(\frac{3}{4}u^{4/3}\right)\right) + C$$

$$= \frac{3}{28}u^{7/3} - \frac{51}{16}u^{4/3} + C$$

$$= \frac{3}{28}(3-2x)^{7/3} - \frac{51}{16}(3-2x)^{4/3} + C$$