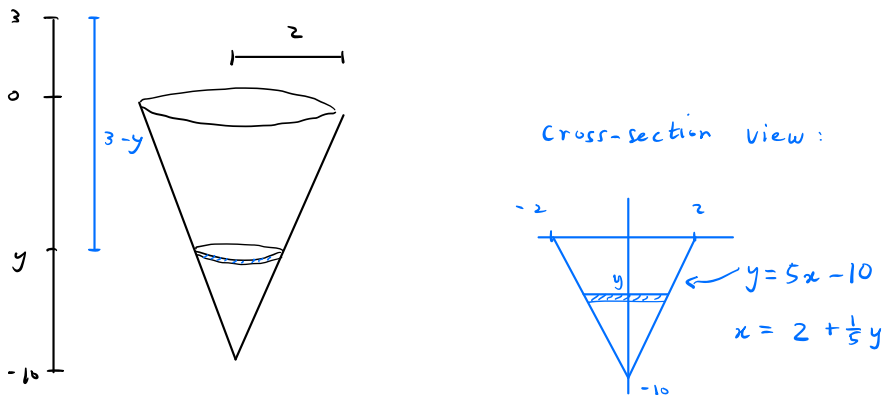


## 7.5 Work, continued

Example A conical tank, shown below, is buried 10 meters below ground, its top is at ground level, and its radius is 2 meters.

It's full of fluid with density 2 kg per cubic meter and we want to find work of pumping all fluid to a height 3 meters above ground.



radius of slice at position  $y$ :

$$r(y) = 2 + \frac{1}{5}y$$

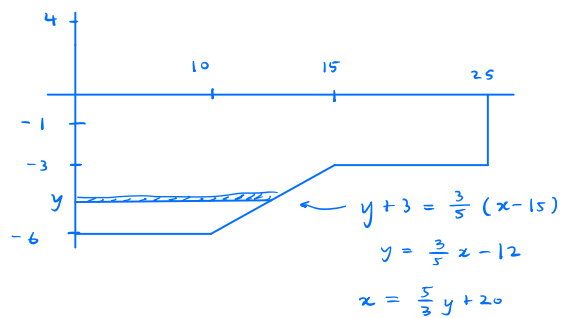
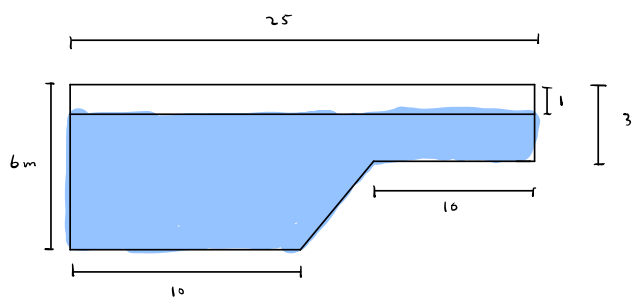
work to lift slice at position  $y$ :

$$(2) \left( \pi \left( 2 + \frac{1}{5}y \right)^2 \Delta y \right) (9.8) (3-y)$$

$\underbrace{\hspace{10em}}_{F = (\text{density})(\text{volume})(\text{accel.})} \quad \underbrace{\hspace{2em}}_d$

$$\text{Total work} = \int_{-10}^0 2\pi(9.8) \left( 2 + \frac{1}{5}y \right)^2 (3-y) dy$$

Example A rectangular tank 20 meters wide is buried with top at ground level. Contains fluid with density 5.2 kg per cubic meter, up to level 1 meter below ground level. Find work to pump 4 meters above ground level. Cross section is shown below.



If  $-6 \leq y \leq -3$ , work to lift slice at  $y$  is

$$5.2 \left( (20) \left( \frac{5}{3}y + 20 \right) \Delta y \right) (9.8) (4 - y)$$

$\underbrace{\hspace{10em}}_{F = (\text{density})(\text{volume})(\text{accl.})} \quad \underbrace{\hspace{2em}}_d$

If  $-3 \leq y \leq -1$ , work to lift slice at  $y$  is

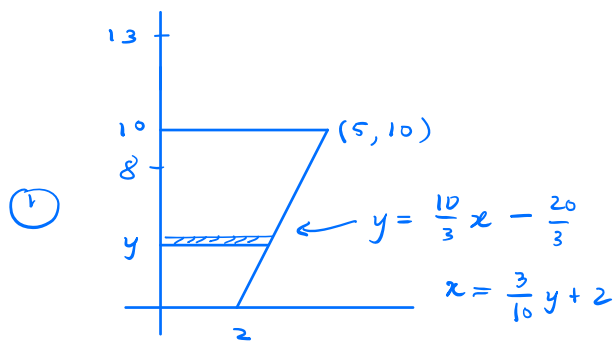
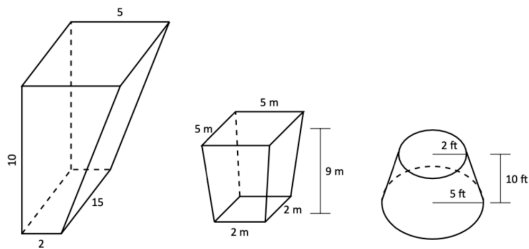
$$5.2 \left( (20)(25) \Delta y \right) (9.8) (4 - y)$$

$\underbrace{\hspace{10em}}_{F = (\text{density})(\text{volume})(\text{accl.})} \quad \underbrace{\hspace{2em}}_d$

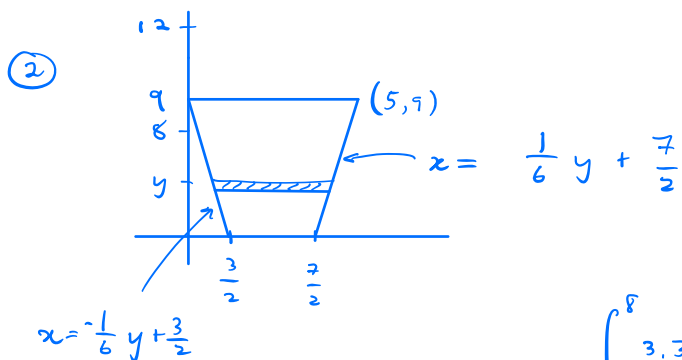
$$\text{Total work} = \int_{-6}^{-3} (5.2)(9.8)(20) \left( \frac{5}{3}y + 20 \right) (4 - y) dy$$

$$+ \int_{-3}^{-1} (5.2)(9.8)(20)(25)(4 - y) dy$$

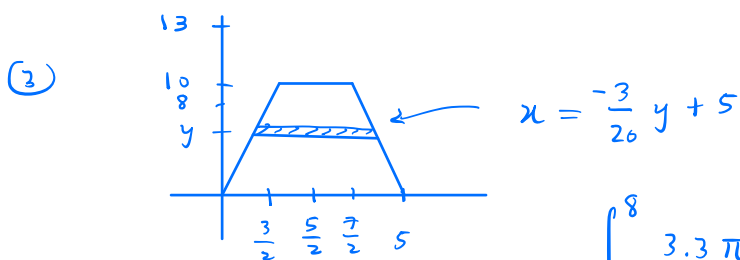
**Problem.** Each of the tanks below is filled to a height of 8 units (assume each figure has lengths given in meters) with a fluid with density 3.3 kilograms per cubic meter. Find the work done in pumping the fluid to a height 3 meters above the top of the tank.



$$\int_0^8 3.3 \left(\frac{3}{10}y + 2\right)(15)(9.8)(13 - y) dy$$



$$\int_0^8 3.3 \left(\frac{1}{6}y + \frac{7}{2}\right)^2 (9.8)(12 - y) dy$$



$$\int_0^8 3.3 \pi \left(-\frac{3}{20}y + \frac{5}{2}\right)^2 (9.8)(13 - y) dy$$