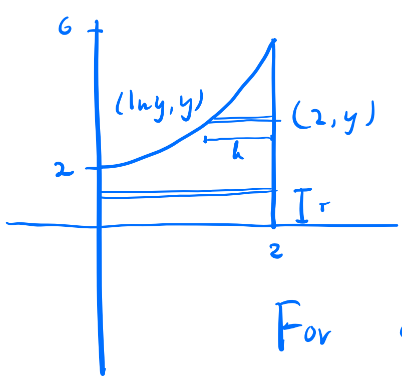
§ 8.2 Volumes

Here's one more practice problem with shells

Example Consider the region between

$y = x^2 + 2$ and $y = 0$

for values of x between 0 and 2 that is rotated about the x-axis. Use shells to set up an integral for its volume.



Our slices have different behavior for $0 \le y \le 2$ compared to $2 \le y \le 6$.

For $0 \le y \le 2$, $r = y$ and $h = 2$

but for $2 \le y \le 6$, $r = y$ and $h = 2 - \ln y$.

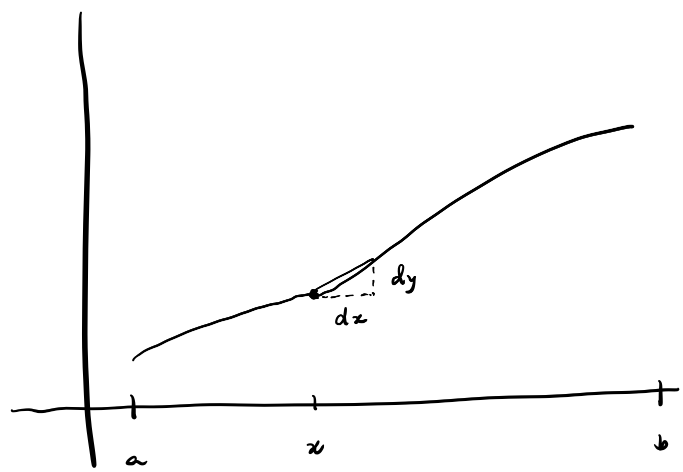
Therefore, the volume is

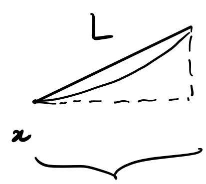
$$V = \int_0^2 2\pi r h \, dy + \int_2^6 2\pi r h \, dy$$

$$= \int_0^2 (2\pi)(y)(2)\, dy + \int_2^6 (2\pi)(y)(2-\ln y)\, dy$$

§8.2 Arclength

How do we find the length of the curve $y = f(x)$ for x from a to b?





$$\underbrace{\quad}_{dx}\Big\} \; dy = f'(x)\,dx$$

Length of small segment $\approx \sqrt{dx^2 + dy^2} = \sqrt{dx^2 + (f'(x)\,dx)^2}$

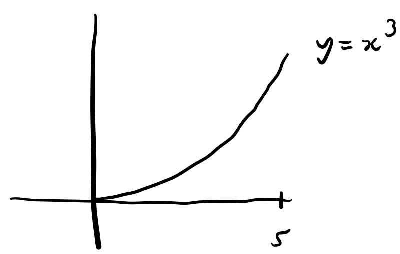
$$= \sqrt{dx^2(1 + f'(x)^2)}$$

$$= \sqrt{1 + f'(x)^2}\,dx$$

Total arc length

$$= \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

<u>Example</u> Find length of $y = x^3$ from $x = 0$ to $x = 5$.



$$\int_0^5 \sqrt{1 + (3x^2)^2}\, dx = \int_0^5 \sqrt{1 + 9x^4}\, dx$$

must use computer to find the value here.

<u>Example</u> Set up integrals for arclengths of the following curves for $x = 0$ to $x = 2$.

1) $f(x) = \frac{x^2}{2}$

2) $f(x) = \ln(x+1)$

3) $f(x) = \sqrt{4 - x^2}$

1) $\int_0^2 \sqrt{1+x^2}\,dx$

2) $\int_0^2 \sqrt{1+\left(\frac{1}{x+1}\right)^2}\,dx$

3) $\int_0^2 \sqrt{1+\left(\frac{x}{\sqrt{4-x^2}}\right)^2}\,dx$