Math 102 — Sequences and Limits

Problem 1. The first few terms of some sequences are written below. Give a general formula s_n for the sequence, assuming the first term corresponds to $n = 1$. Conclude by stating whether each sequence converges or diverges. No need to give justification.

a. $1, -1, 1, -1, 1, -1, \ldots$

b. $\frac{1}{5}, \frac{-1}{6}, \frac{1}{7}, \frac{-1}{8}, \frac{1}{9}, \frac{-1}{10}, \ldots$

c. $\frac{1}{5}, \frac{-3}{6}, \frac{5}{7}, \frac{-7}{8}, \frac{9}{9}, \frac{-11}{10}, \ldots$

Problem 2. Consider the limits below. We have learned a quick rule to compute limits like these quickly by looking at leading powers and leading coefficients. However, I'd like you to compute these by (1) multiplying the numerator and denominator by a fraction of the form $\frac{1}{n^k}$, (2) distributing, and then (3) computing limits term by term. This will help you internalize the reasoning behind the shortcut.

a. $\displaystyle\lim_{n\to\infty} \frac{3n^2 + 2n + 5}{7n^2 + 3n + 1}$

b. $\displaystyle\lim_{n\to\infty} \frac{5n^3 + 2n + 4}{9n^2 + 11n + 1}$

c. $\displaystyle\lim_{n\to\infty} \frac{4n^2 + 2n + 5}{3n^5 + 3n + 1}$

Problem 3. Consider the four sequences below. Two of them converge (their limits are both 0). One of them diverges toward $+\infty$ (we say it diverges but its limit is $+\infty$). One of them diverges (it does not tend toward a single finite or infinite value; it has no limit). Write out the first five terms of each sequence (starting from $n = 1$) and identify which behavior each has.

a. $a_n = 2^n$

b. $b_n = \left(\frac{1}{2}\right)^n$

c. $c_n = (-2)^n$

d. $d_n = \left(-\frac{1}{2}\right)^n$