Math 102 — Ratio test

Summary. Try each of the following problems together in a small group.

Problem 1. What does the ratio test tell us about each of the following series?

a. $\sum_{n=1}^{\infty} \frac{1}{n3^n}$

b. $\sum_{n=1}^{\infty} \frac{4^n}{n!}$

c. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n!}$

d. $\sum_{n=1}^{\infty} \frac{(n-1)!}{n^2}$

e. $\sum_{n=1}^{\infty} \frac{1}{n}$

f. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

(a) $$\lim_{n\to\infty} \frac{n \cdot 3^n}{(n+1)3^{n+1}} = \lim_{n\to\infty} \frac{n}{n+1} \cdot \frac{3^n}{3^{n+1}}$$

$$= \lim_{n\to\infty} \frac{n}{n+1} \cdot \frac{1}{3}$$

$$= \frac{1}{3} < 1 \quad \text{(converges)}$$

(b) $$\lim_{n\to\infty} \frac{4^{n+1}}{(n+1)!} \frac{n!}{4^n} = \lim_{n\to\infty} \frac{4^{n+1}}{4^n} \cdot \frac{n!}{(n+1)!}$$

$$= \lim_{n\to\infty} 4 \cdot \frac{1}{n+1} = 0 < 1 \quad \text{(converges}$$

(c) $$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(n+1)^2}{(n+1)!} \frac{n!}{(-1)^n \cdot n^2} \right|$$

$$= \lim_{n\to\infty} \frac{n!}{(n+1)!} \frac{(n+1)^2}{n^2}$$

$$= \lim_{n\to\infty} \frac{1}{n+1} \frac{(n+1)^2}{n^2} = 0 < 1$$ (converges)

(d) $$\lim_{n\to\infty} \frac{n!}{(n+1)^2} \frac{n^2}{(n-1)!}$$

$$= \lim_{n\to\infty} \frac{n^2}{(n+1)^2} \frac{n!}{(n-1)!}$$

$$= \lim_{n\to\infty} \frac{n^3}{(n+1)^2} = \infty > 1$$ (diverges)

(e) $$\lim_{n\to\infty} \frac{n}{n+1} = 1$$ (inconclusive, but we know it diverges since it's a p-series with $p=1$)

(f) $$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n} \right| = \lim_{n\to\infty} \frac{n^2}{(n+1)^2} = 1$$

(inconclusive, but it converges by absolute convergence test since $\sum \left| \frac{(-1)^n}{n^2} \right| = \sum \frac{1}{n^2}$

is a p-series with $p = 2 > 1$)