Math 102 — Geometric series

Problem 1. For each infinite series below, determine whether it is a geometric series. *Hint*: is there a common ratio between successive terms? If it is geometric, identify the values of a and r. In fact, each series converges. Find the value that each is equal to. Make sure to discuss any subtleties you notice with groupmates.

a. $(1/3)^4 + (1/3)^5 + (1/3)^6 + \cdots$

b. $5 - \frac{5}{4} + \frac{5}{4^2} - \frac{5}{4^3} + \cdots$

c. $20 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} + \cdots$

Problem 2. For each example below, we will treat a_n in two different ways: (1) as the terms of a sequence, and (2) as the terms of a series. Find $\lim_{n\to\infty} a_n$ if the *sequence* converges and find $\sum_{n=0}^{\infty} a_n$ if the *series* converges.

a. $a_n = \left(\frac{5}{6}\right)^n$

b. $a_n = \left(-\frac{3}{4}\right)^n$

Problem 3. For each example below, follow the same instructions as the previous problem. Note that these are straightforward to study as terms of a sequence, but tricky as terms of a series. They are not terms of a geometric series. Can you explain why not? You'll probably feel unsure of how to handle these as terms of a series, but I want you to have a discussion with groupmates and come up with some guesses or conjectures based on any intuition you can gather.

a. $a_n = \frac{1}{2}$

b. $a_n = \frac{2n^3+n+3}{5n^3+n^2+4n+6}$

c. $a_n = \frac{1}{n}$