Math 102 — Sequences

Summary. Try each of the following problems together in a small group.

Problem 1. Determine whether each of the following sequences converges. If it does, find its limit. If it doesn't, explain how it diverges (ie. grows to $+\infty$, decreases to $-\infty$, or oscillates between various values).

a. $4^n/5^n$

b. $\frac{1}{n} + \sin(\pi n)$

c. $\frac{5n^4+2n^2+3}{3n^4+n^3+n}$

d. $\frac{n^2}{3} + \frac{3}{n^2} - 3^n$

Problem 2. Decide whether each of the following statements is true or false. Give an explanation for your answer.

a. You can tell a sequence converges by looking at the first 1000 terms.

b. If the terms s_n of a convergent sequence are all positive, then $\lim_{n\to\infty} s_n$ is positive.

c. If a sequence of positive terms is unbounded, then the sequence has a term greater than a million.

Problem 3. In our textbook, the term *bounded sequence* is defined to be a sequence where all the terms are trapped between two constants. In symbols, we say a sequence is bounded if we can find some constants A and B where $A \leq s_n \leq B$ for all $n \geq 1$.

a. Can you think of a bounded sequence that fails to converge?

b. Is it possible to have a bounded sequence that's *increasing*? That is, is it possible to have a bounded sequence where the terms keep getting bigger as we move along the list of terms? What would a plot of a bounded sequence look like if we plotted it on the xy-plane? Can you think of a real world example of a bounded increasing sequence that involves, say, year-to-year salaries for a certain job?

c. What general conclusion can you make about sequences that are bounded and increasing?

d. What do you think can be said in general about bounded sequences that are *decreasing*?