Math 102 — Integration by substitution

Summary. Try each of the following problems together in a small group.

Problem 1. Find the following antiderivatives using u-substitution. Begin by identifying u and compute $du = u'(x)\,dx$.

a. $\int e^{-4x}\,dx$

b. $\int \sin(2x)\,dx$

c. $\int x^2 e^{x^3+1}\,dx$

d. $\int x(x^2+3)^2\,dx$

e. $\int \frac{\sin\theta}{\cos\theta+5}\,d\theta$

f. $\int \tan\theta\,d\theta$

a) $\int e^{-4x}\,dx$ $\qquad u = -4x$

$du = -4dx$

$= -\frac{1}{4}\int e^u\,du$ $\qquad -\frac{1}{4}du = dx$

$= -\frac{1}{4}e^u + C = -\frac{1}{4}e^{-4x} + C$

b) $\int \sin(2x)\,dx$ $\qquad u = 2x$

$du = 2dx$

$= \frac{1}{2}\int \sin(u)\,du$ $\qquad \frac{1}{2}du = dx$

$$= \frac{-1}{2}\cos(u) + C = \frac{-1}{2}\cos(2x) + C$$

c) $\int x^2 e^{x^3+1}\,dx$

$u = x^3 + 1$

$du = 3x^2\,dx$

$= \frac{1}{3}\int e^u\,du$

$\frac{1}{3}du = x^2\,dx$

$$= \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3+1} + C$$

d) $\int x(x^2+3)^2\,dx$

$u = x^2 + 3$

$du = 2x\,dx$

$= \frac{1}{2}\int u^2\,du$

$\frac{1}{2}du = x\,dx$

$$= \frac{1}{2}\left(\frac{1}{3}u^3\right) + C$$

$$= \frac{1}{6}(x^2+3)^3 + C$$

e) $\int \frac{\sin\theta}{\cos\theta + 5}\,d\theta$

$u = \cos\theta + 5$

$du = -\sin\theta\,d\theta$

$= -\int \frac{1}{u}\,du$

$-du = \sin\theta\,d\theta$

$$= -\ln|u| + C = -\ln|\cos\theta + 5| + C$$

f)

$$\int \tan\theta \, d\theta = \int \frac{\sin\theta}{\cos\theta} \, d\theta \qquad u = \cos\theta$$

$$du = -\sin\theta \, d\theta$$

$$= \int \frac{-1}{u} \, du \qquad -du = \sin\theta \, d\theta$$

$$= -\ln|u| + C$$

$$= -\ln|\cos\theta| + C$$