

## Math 102 — Integration by substitution

*Summary.* Try each of the following problems together in a small group.

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**Problem 1.** Find the following definite integrals using  $u$ -substitution and changing the limits of integration.

a.  $\int_{-1}^2 \sqrt{x+2} dx$

b.  $\int_0^\pi \cos(x+\pi) dx$

c.  $\int_0^{\pi/2} e^{-\cos\theta} \sin\theta d\theta$

d.  $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

a)  $u = x+2$   
 $du = dx$

$$\int_1^4 u^{1/2} du = \frac{2}{3} u^{3/2} \Big|_1^4$$
$$= \frac{2}{3} (4^{3/2} - 1) = \frac{14}{3}$$

b)  $u = x+\pi$   
 $du = dx$

$$\int_\pi^{2\pi} \cos(u) du$$
$$= \sin u \Big|_\pi^{2\pi} = 0$$

c)  $u = -\cos\theta$   
 $du = \sin\theta d\theta$

$$\int_{-1}^0 e^u du = e^u \Big|_{-1}^0 = 1 - e^{-1}$$

$$\begin{aligned} u) \quad u &= \sqrt{x} \\ du &= \frac{1}{2} x^{-1/2} dx \\ 2du &= x^{-1/2} dx \end{aligned}$$

$$\begin{aligned} & 2 \int_1^2 e^u du \\ &= 2e^u \Big|_1^2 = 2(e^2 - e) \end{aligned}$$