Math 102 — Integration by parts

Summary. Try each of the following problems together in a small group.

Problem 1. Find the following definite integrals using integration by parts. Remember to start by making a choice for u and for dv. Through practice, these choices will get easier, but we often choose u to be something that gets "simpler" when it's differentiated (like a how polynomial goes to a lower degree when differentiated).

a. $\int x \sin x \, dx$

b. $\int xe^{5x} \, dx$

c. $\int x^5 \ln(4x) \, dx$

d. $\int x^3 e^{x^2} \, dx$ *Hint: try doing a substitution first. Use the letter y instead of u when doing your substitution.*

e. $\int x^2 e^{4x} \, dx$ *Hint: you'll need to do integration by parts twice.*

a) $u = x \qquad dv = \sin x \, dx$

$du = dx \qquad v = -\cos x$

$= -x\cos x + \int \cos x \, dx$

$= -x\cos x + \sin x + C$

b) $u = x \qquad dv = e^{5x}\,dx$

$du = dx \qquad v = \frac{1}{5}e^{5x}$

$$= \frac{1}{5}xe^{5x} - \int \frac{1}{5}e^{5x}\,dx$$

$$= \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + C$$

c) $u = \ln(4x) \qquad dv = x^5\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{6}x^6$

$$= \frac{1}{6}x^6\ln(4x) - \frac{1}{6}\int x^5\,dx$$

$$= \frac{1}{6}x^6\ln(4x) - \frac{1}{36}x^6 + C$$

d) $y = x^2$

$dy = 2x\,dx \qquad \frac{1}{2}dy = x\,dx$

$$= \int x^2 \cdot x\,e^{x^2}\,dx$$

$$= \int \tfrac{1}{2}y e^{y}\,dy \qquad u = \tfrac{1}{2}y \qquad dv = e^y\,dy$$

$du = \frac{1}{2}dy \qquad v = e^y$

$$= \frac{1}{2} y e^y - \frac{1}{2} \int e^y dy$$

$$= \frac{1}{2} y e^y - \frac{1}{2} e^y + C$$

e) $u = x^2 \qquad dv = e^{4x} dx$

$du = 2x dx \qquad v = \frac{1}{4} e^{4x}$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{2} \int x e^{4x} dx \qquad u = x \quad dv = e^{4x} dx$$

$du = dx \quad v = \frac{1}{4} e^{4x}$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{2} \left(\frac{1}{4} x e^{4x} - \frac{1}{4} \int e^{4x} dx \right)$$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{2} \left(\frac{1}{4} x e^{4x} - \frac{1}{16} e^{4x} \right) + C$$

$$= \frac{1}{4} x^2 e^{4x} - \frac{1}{8} x e^{4x} + \frac{1}{32} e^{4x} + C.$$