Math 102 — More integration by parts

Summary. Try each of the following problems together in a small group.

Problem 1. Find the following definite integrals using integration by parts. Remember to start by making a choice for u and for dv. Sometimes you might have to use substitution first or do integration by parts twice or do some algebra after using integration by parts.

a. $\int x^2 \sin x \, dx$

b. $\int \sin^2 x \, dx$

c. $\int x^2 \cos(x^3) \, dx$

Problem 2. Find the following areas.

a. Under $y = xe^{-x}$ for $0 \leq x \leq 2$.

b. Between $y = \ln x$ and $y = \ln(x^2)$ for $1 \leq x \leq 2$.

Problem 1

(a) $u = x^2 \qquad dv = \sin x \, dx$

$du = 2x \, dx \qquad v = -\cos x$

$= -x^2 \cos x + \int 2x \cos x \, dx$ $\qquad u = 2x \qquad dv = \cos x \, dx$

$du = 2dx \qquad v = \sin x$

$= -x^2 \cos x + 2x \sin x - \int 2 \sin x \, dx$

$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$

(b) $u = \sin x \qquad dv = \sin x\,dx$

$du = \cos x\,dx \qquad v = -\cos x$

$$= -\sin x \cos x + \int \cos^2 x\,dx$$

$$= -\sin x \cos x + \int (1 - \sin^2 x)\,dx$$

$$= -\sin x \cos x + x - \int \sin^2 x\,dx$$

$$2\int \sin^2 x\,dx = -\sin x \cos x + x$$

$$\int \sin^2 x\,dx = -\frac{1}{2}\sin x \cos x + \frac{1}{2}x + C$$

(c) no integration by parts:

$u = x^3$

$du = 3x^2\,dx$

$\frac{1}{3}\,du = x^2\,dx$

$$= \frac{1}{3}\int \cos u\,du$$

$$= \frac{1}{3}\sin u + C$$

$$= \frac{1}{3}\sin(x^3) + C$$

Problem 2

(a)

$$\int_0^2 xe^{-x}\,dx \qquad u = x \quad dv = e^{-x}dx$$

$$du = dx \quad v = -e^{-x}$$

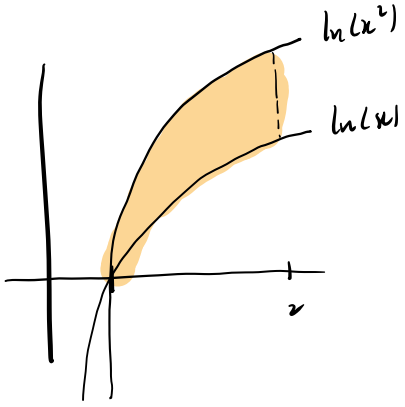
$$= -xe^{-x}\Big|_0^2 + \int_0^2 e^{-x}dx$$

$$= -2e^{-2} + \left(-e^{-x}\Big|_0^2\right)$$

$$= -2e^{-2} - e^{-2} + 1$$

$$= 1 - 3e^{-2}$$

(b)



$$\int_1^2 [\ln(x^2) - \ln(x)]\,dx$$

$$= \int_1^2 [2\ln x - \ln x]\,dx$$

$$= \int_1^2 \ln x\,dx$$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

$$= x\ln x\Big|_1^2 - \int_1^2 dx = 2\ln 2 - 1$$