

Math 102 — More Taylor series practice

Problem 1. Use Maclaurin series to evaluate the following limits:

- a. $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2}$
- b. $\lim_{x \rightarrow 0} \frac{\ln(1-x^2)}{x^2}$
- c. $\lim_{x \rightarrow 0} \frac{e^{x^2} - x^2 - 1}{x^4}$
- d. $\lim_{x \rightarrow 0} \frac{\cos(\sqrt{x}) - 1}{2x}$

Problem 2. Find the radius of convergence of the Maclaurin series for $f(x) = 1/(1 - 2x)$.

Problem 3. Suppose $f(0) = 0, f'(0) = 1, f''(0) = -3, f'''(0) = 7, f^{(4)}(0) = -15$. Estimate the value of $\int_0^{0.6} f(x) dx$.

Problem 4. Suppose f is infinitely differentiable at 0 and its Maclaurin series is given by

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$$

Find $f'(0), f''(0), f'''(0)$, and $f^{(10)}(0)$.

Problem 5. Suppose x is a positive but very small number. Arrange the following expressions in increasing order:

$$x, \sin x, \ln(1+x), 1 - \cos x, e^x - 1, \arctan x.$$