Math 102 — Exam 3 review

Your third exam will take place during the self-scheduled exam period and will contain about 6 problems, some with multiple parts. It will cover material from Homework 8 to Homework 10. Nearly all questions are directly related to material since the last exam. There is a cumulative aspect to these questions due to the nature that recent material builds on material from earlier in the semester. There will also be a problem that asks you to compute some integrals using techniques we've learned. The problems below give you a sampling of some similar problems, but it's not necessarily comprehensive, so make sure to review old homework, worksheets, and lecture notes. There are also problems in our textbook, with answers to odd-numbered problems in the back. No notes will be allowed on the exam, but you can use a scientific calculator with no graphing functionality.

Problem 1. Find the interval of convergence of the following power series.

a. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{n^2 5^n}$

b. $\displaystyle\sum_{n=1}^{\infty}\frac{(x+2)^n}{n4^n}$

c. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n(x-2)^n}{(n+2)!}$

Problem 2. Find the Maclaurin series each of the following functions.

a. $f(x)=\sin(x^4)$

b. $f(x)=xe^{2x}$

c. $f(x)=\ln(1-x^3)$

d. $f(x)=\dfrac{x^3}{(1+x)^2}$

e. $f(x)=x^4+4x^3+5x^2-3x-7$

Problem 3. Find the sum of the series below.

a. $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!4^n}$

b. $\sum_{n=0}^{\infty}(-1/4)^n$

c. $\sum_{n=1}^{\infty}n(-1/4)^{n-1}$

d. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n+1}}{4^{2n+1}(2n+1)!}$

e. $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{9^n(2n)!}$

Problem 4. A variety of tanks filled with oil, which has density 800 kg per cubic meter, are given below. Each tank is only filled to half the tank's height. Set up but do not compute an integral for the work performed in pumping all the oil to a height 3 meters above the top of the tank.

a. An upright circular cylinder with height 4 m and base radius 3 m.

b. A cone whose base circle has radius 3 m and whose height is 10 m, oriented so that its tip is at the bottom of the tank.

c. A pyramid whose base is a square with side length 4 m and whose height is 7 m, oriented so that its tip is at the bottom of the tank.

Problem 5. Use Taylor's theorem to find a bound for the error in approximating the given quantity with a third degree Maclaurin polynomial for the given function.

a. $\sin(0.2), f(x) = \sin x$

b. $\sqrt{0.9},\ f(x) = \sqrt{1+x}$

c. $1/\sqrt{3}, f(x) = (1+x)^{-1/2}$

Problem 6. Find the Taylor polynomial of degree 4 of $f(x)$ centered at c for the given examples below.

a. $f(x) = 1/(1+x), c = 2$

b. $f(x) = \sin x, c = -\pi/4$

c. $f(x) = \ln(x^2), c = 1$