Math 102 — Convergence of series

Summary. Try each of the following problems together in a small group.

Problem 1. Use the integral test to decide whether each of the following series converges.

a. $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$

b. $\sum_{n=1}^{\infty} \frac{1}{n^{0.9}}$

c. $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$

Problem 2. What do you suspect happens with $\sum_{n=1}^{\infty} \frac{1}{n^{p}}$ in general? For which values of p do we get convergence? For which values do we get divergence?

Problem 3. For each of the following series, identify whether it converges or diverges.

a. $\sum_{n=1}^{\infty} \frac{3n^2+5}{4n^2+10}$

b. $\sum_{n=1}^{\infty} \frac{5n}{6n^2}$

c. $\sum_{n=1}^{\infty} e^n$

d. $\sum_{n=1}^{\infty} \frac{n+2^n}{n2^n}$

e. $\sum_{n=1}^{\infty} \frac{1}{n^2} + (1/3)^n$

f. $\sum_{n=1}^{\infty} (0.5)^n + (-1)^n$

Problem 4. When talking about limits of sequences in the first couple of days in class, we talked about how the leading terms in a complicated limit like

$$\lim_{n\to\infty} \frac{n^2+n+1}{n^5+n^2+3}$$

are the important factors. When we look at the limit like the one above, we think: "that limit basically behaves like $1/n^3$." Use this intuition to make conjectures about whether the following series converge.

a. $\sum_{n=1}^{\infty} \frac{n^4+3n^2+5}{n^5+n^4+4n}$

b. $\sum_{n=1}^{\infty} \frac{4}{(n-1)^3}$

c. $\sum_{n=1} \frac{4n^2+n}{5n^4}$