

Math 102, Fall 2021 — Exam 1 notes

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Exam 1 on October 8

Remarks

Your exam in class on October 8 will contain about 6 problems, some with multiple parts. It will cover sections 9.1 to 9.4 in our textbook (sequences, geometric series, p -series, and all the convergence tests). I won't ask you to use the integral test since we haven't practiced very much integration yet. The problems will all be similar to homework. While old worksheets, lecture problems, homework problems all give lots of examples (try them without looking at solutions), there are also problems in our textbook, with answers to odd-numbered problems in the back. No notes will be allowed on the exam, but you can use a scientific calculator with no graphing functionality.

The sample problems below give you some practice of the main topics, but make sure to study old problems to get a complete overview.

Sample problems

1. Find a formula for the general term a_n of the following sequences; assume the sequence starts with index $n = 1$.

(a) $-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \dots$

(b) $\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \dots$

2. Find the limit of the following sequences or state that it does not exist.

(a) $a_n = n^2 2^{-n}$

(b) $a_n = \frac{n^3 + 4n^2 + 3}{8n^3 + 5n + 7}$

(c) $a_n = \frac{9^{n+1}}{10^n}$

(d) $a_n = (-1)^n \frac{n^4 + 1}{3n^4 + 4}$

(e) $a_n = \frac{n \cos n}{n^2 + 1}$

3. Find the sum of the infinite series

(a) $-3 + 2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \dots$

(b) $\sum_{n=1}^{\infty} 6(0.9)^{n-1}$

(c) $\sum_{n=1}^{\infty} \frac{(-3)^n}{4^n}$

4. Maintenance on a new car starts at \$500 by the end of the first year of ownership but increases by 20% annually. Find the total maintenance cost after 15 years of ownership.

5. Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges where a_n is given as follows:

(a) $a_n = \frac{1}{n\sqrt{n^2+1}}$

(b) $a_n = \frac{n!}{5^n}$

(c) $a_n = \frac{n}{3n+1}$

(d) $a_n = (-1)^n \frac{1}{5n^2+1}$

(e) $a_n = (-1)^{n-1} \frac{1}{\sqrt{5n^2+1}}$

(f) $a_n = \frac{3}{n+(1.2)^n}$

(g) $a_n = \frac{2^n n!}{(n+2)!}$

(h) $a_n = \frac{n^{0.1}-1}{n(\sqrt{n+1})}$

6. Determine whether the following series converge absolutely, converge conditionally, or diverge.

(a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^5+2}$

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{5n+2}}$

(c) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^{1/3}+3}$