Math 203, Spring 2023 — Homework 1

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Due February 3

Instructions. This problem set has material from Week 1 of class.

Problem 1. Let $P = (1, 0, 2)$, $Q = (3, 4, -1)$, and $\mathbf{v} = \overrightarrow{PQ}$.

a. Write $\mathbf{v}$ in component form.

b. Find the magnitude of $\mathbf{v}$.

c. Find the unit vector that points in the direction opposite of $\mathbf{v}$.

d. Find the vector of length 3 that points in the direction of $\mathbf{v}$.

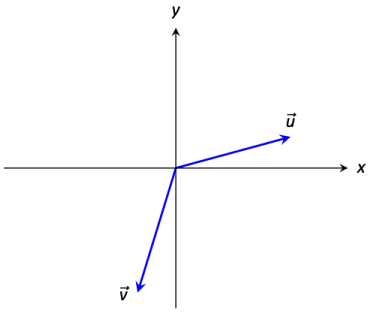
Problem 2. Let $\mathbf{u} = \langle 1, 1, -1 \rangle$ and $\mathbf{v} = \langle 2, 1, 2 \rangle$.

a. If $\mathbf{u} = \overrightarrow{PQ}$ with basepoint $P = (2, 2, 2)$, what must Q be?

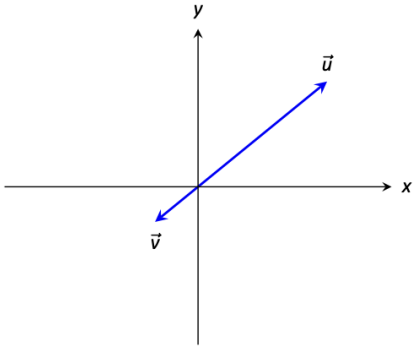
b. Find $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.

c. Explain whether $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ are parallel using algebra.

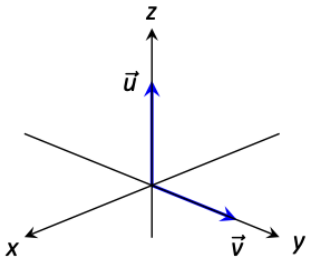
Problem 3. For each part below, sketch $\mathbf{u}, \mathbf{v}, \mathbf{u}+\mathbf{v}, \mathbf{u}-\mathbf{v}$ on the same set of axes.



a.



b.



c.

Problem 4. Which of the following pairs of vectors are orthogonal.

a. $\mathbf{v} = \langle 5, 3 \rangle, \mathbf{w} = \langle 6, 1 \rangle$

b. $\mathbf{v} = \langle 4, 7 \rangle, \mathbf{w} = \langle 7, -4 \rangle$

c. $\mathbf{v} = \langle 3, 5, -1 \rangle, \mathbf{w} = \langle 4, -1, 8 \rangle$

d. $\mathbf{v} = \langle 1, 2, 3 \rangle, \mathbf{w} = \langle 0, 0, 0 \rangle$

Problem 5. Let $\mathbf{v} = \langle 5, 5 \rangle, \mathbf{w} = \langle 1, 3 \rangle$, and θ the angle between them. Find the following:

a. $\cos\theta$.

b. $\text{proj}_{\mathbf{v}} \mathbf{w}$.

c. $\text{proj}_{\mathbf{w}} \mathbf{v}$.

d. Sketch the vectors $\mathbf{v}, \mathbf{w}, \text{proj}_{\mathbf{v}}\mathbf{w}$ on the same axes, all with the same base point.

Problem 6. Let $\mathbf{v} = \langle 1, 0, 1 \rangle, \mathbf{w} = \langle 1, 1, 1 \rangle$, and θ the angle between them. Find the following:

a. $\cos\theta$.

b. $\text{proj}_{\mathbf{v}} \mathbf{w}$.

c. $\text{proj}_{\mathbf{w}} \mathbf{v}$.

Problem 7. Let $\mathbf{u} = \langle 1, 4, 1 \rangle$ and $\mathbf{v} = \langle -1, 3, 5 \rangle$. Use orthogonal projection to find a vector $\mathbf{p}$ that is parallel to $\mathbf{v}$ and a vector $\mathbf{x}$ that is orthogonal to $\mathbf{v}$ which satisfy the equation $\mathbf{u} = \mathbf{p} + \mathbf{x}$.