Math 203, Spring 2023 — Homework 2

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Due February 10

Instructions. This problem set has material from Week 2 of class.

Problem 1. Let $\mathbf{v} = \langle 3, 2, -2\rangle$ and $\mathbf{w} = \langle 0, 1, 5\rangle$. Compute the areas of the following shapes.

a. parallelogram determined by $\mathbf{v}$ and $\mathbf{w}$.

b. triangle determined by $\mathbf{v}, \mathbf{w}$, and $\mathbf{w} - \mathbf{v}$.

Problem 2. Let $\mathbf{v} = \langle 1, 1, 2\rangle$ $\mathbf{w} = \langle 2, 0, 3\rangle$, and θ the angle between them. Compute the following quantities.

a. $\mathbf{v} \times \mathbf{w}$

b. $\mathbf{w} \times \mathbf{v}$

c. $\sin\theta$

Problem 3. Write vector equations of the lines given by the following descriptions.

a. Passing through $P = (6, 1, 7)$ and parallel to $\mathbf{v} = \langle -3, 2, 5\rangle$.

b. Passing through the points $P = (1, -1, 1)$ and $Q = (4, 0, 1)$.

c. Passing through the point $P = (0, 1, 2)$ and orthogonal to both $\mathbf{v} = \langle 2, -1, 7\rangle$ and $\mathbf{w} = \langle 7, 1, 3\rangle$.

Problem 4. The following two lines intersect:

$$\begin{aligned} \ell_1(t) &= \langle t, -2 + 2t, 1 + t\rangle \\ \ell_2(t) &= \langle 2 + t, 2 - t, 3 + 2t\rangle . \end{aligned}$$

Find their point of intersection by solving a system of three equations with two unknowns.

Problem 5. Consider the following two lines:

$$\begin{aligned} \ell_1(t) &= \langle 1 + 2t, 3 - 2t, t\rangle \\ \ell_2(t) &= \langle 3 - t, 3 + 5t, 2 + 7t\rangle . \end{aligned}$$

Determine whether they are the same line, parallel lines, intersecting lines, or skew lines.

Problem 6. Find standard form equations of the two planes with the following descriptions.

a. Contains the point $(4, 5, 6)$ and is orthogonal to the line $\ell(t) = \langle -5 + 4t, 2 + t, 3 - 2t \rangle$.

b. Contains the point $(-4, 7, 2)$ and is parallel to the plane $3(x - 2) + 8(y + 1) - 10z = 0$.

Problem 7. Find standard form equations of the two planes with the following descriptions.

a. The plane that contains the intersecting lines $\ell_1(t) = \langle 5, 0, 3 \rangle + t \langle -1, 1, 1 \rangle$ and $\ell_2(t) = \langle 1, 4, 7 \rangle + t \langle 3, 0, -3 \rangle$. Use $(x_0, y_0, z_0) = (5, 0, 3)$.

b. The plane that contains the parallel lines $\ell_1(t) = \langle 1, 1, 1 \rangle + t \langle 1, 2, 3 \rangle$ and $\ell_2(t) = \langle 1, 1, 2 \rangle + t \langle 1, 2, 3 \rangle$. Use $(x_0, y_0, z_0) = (1, 1, 1)$. *Hint:* find a vector whose basepoint is on ℓ_1 and whose tip is on ℓ_2.

Problem 8. Consider the line $\ell(t) = \langle 4, 1, 0 \rangle + t \langle 1, 0, -1 \rangle$ and the plane $3x + y - 2z = 8$.

a. Give a brief explanation for why the line and the plane are not parallel.

b. Find the point of intersection between the line and the plane.