Math 203, Spring 2023 — Homework 5

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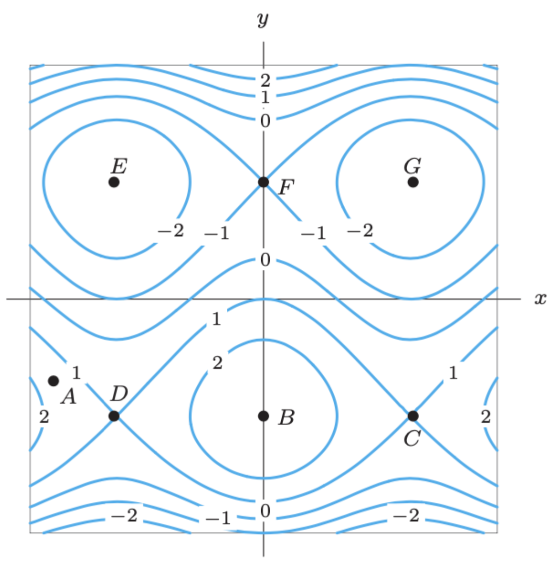
Due March 10

Instructions. This problem set has material from Week 6 of class.

Problem 1. Consider the contour plot below, which contains the points A, B, C, D, E, F, G. For each point, classify it as a *local minimum*, a *local maximum*, a *saddle point*, or *neither*.

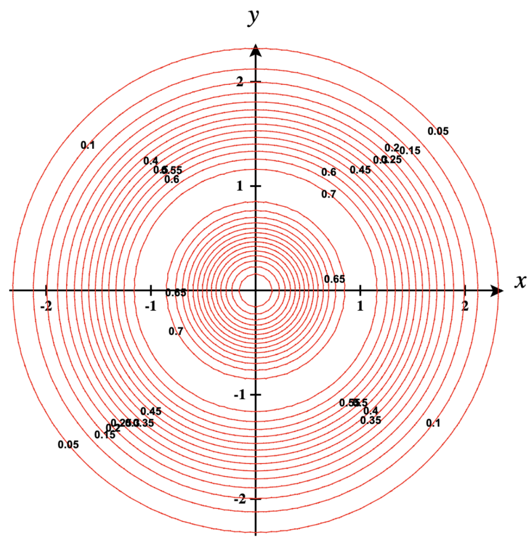


Point	Classification
A	
B	
C	
D	
E	
F	
G	

Problem 2. Find the critical points of the function $f(x, y) = \frac{1}{2}x^2 + 3y^3 + 9y^2 - 3xy + 9y - 9x$ and, if possible, classify them using the Second Derivative Test.

Problem 3. Find the critical points of the function $f(x, y) = x^3 + y^3 - 6y^2 - 3x + 9$ and, if possible, classify them using the Second Derivative Test.

Problem 4. Find all critical points of the function $f(x, y) = 2(x^2 + y^2)e^{-(x^2+y^2)}$. Note this function has infinitely many critical points. After doing some computation, you should be able to the set of all critical points in the xy-plane with an equation or with a short description in words. You'll also find that the Second Derivative Test is inconclusive. Use the contour plot below, along with CalcPlot3D, to explain whether your critical points are local minima or local maxima.



Problem 5. Let $f(x, y) = x^2 + y^2 + y + 1$. Find the global minimum and maximum of f subject to the constraint that (x, y) must come from the region with boundary given by the triangle with vertices $(0, 1)$, $(-1, -1)$, and $(1, -1)$ and the interior of this triangle.

Problem 6. Let $f(x, y) = 5x - 7y$. Find the global minimum and maximum of f subject to the constraint that (x, y) must come from the region with boundary given by the curves $y = x^2$ and $y = 1$ and the interior of this region.

Problem 7. Use the method of Lagrange multipliers to find maximum and minimum values of $f(x, y) = xy$ subject to the constraint that $4x^2 + 8y^2 = 16$.

Problem 8. A rectangular box with a square bottom and no top is to be made with 12 square meters of cardboard. That is, the box has an outer surface area of 12 square meters. Use the method of Lagrange multipliers to find the maximum volume of such a box.

Problem 9. Use the method of Lagrange multipliers to find two non-negative numbers whose sum is 6 such that the sum of their squares is as small as possible. Further, what are the smallest and largest possible sum of squares under these constraints?

Problem 10. In economics, the Cobb-Douglas production function is used to model the production level of a firm given levels of input like labor and capital. A company has determined that its production level is given by the Cobb-Douglas function $f(x, y) = 2.5x^{0.45}y^{0.55}$ where x represents the total number of labor hours in 1 year and y represents the total capital input for the company in 1 year. Suppose 1 unit of labor costs \$40 and 1 unit of capital costs \$50. Use the method of Lagrange multipliers to find the maximum value of f subject to the budgetary constraint of \$500,000 per year.