Math 203, Spring 2026 — Homework 5

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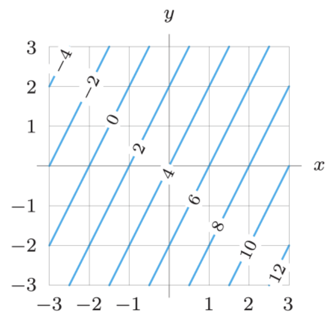
Due March 13

Instructions. This problem set contains problems from Week 6 of class. The problem numbers refer to the PDF version of our textbook, *Apex Calculus*, by Gregory Hartman.

Problem 1. Below is a contour diagram for the linear function (ie. plane) $L(x, y) = mx + ny + b$. Determine the values of m, n, and b.



Problem 2. Find the equation of the tangent plane to the graph of $f(x, y) = ye^{x/y}$ at $(1, 1)$. Write your equation in the form $z = z_0 + m(x - x_0) + n(y - y_0)$ for some constants m, n, x_0, y_0, and z_0.

Problem 3. An unevenly heated 2-dimensional metal plate has temperature $T(x, y)$ in degrees Celcius at any point (x, y) on the plate. Given that $T(2, 1) = 135, T_x(2, 1) = 16$, and $T_y(2, 1) = -15$, estimate the temperature at the point $(2.04, 0.97)$.

Problem 4. Ocean wave heights are influenced by both wind speed and duration. Let $h(v, t)$ represent the wave height (in feet), where v is the wind speed (in knots), and t is the duration (in hours) that the wind has maintained that speed. A table of sample values for $h(v, t)$ is given below. Using this data, construct a local linear approximation for $h(v, t)$ near the point $(40, 20)$. Then apply your approximation to estimate the wave height when the wind has been blowing at 43 knots for 24 hours.

Duration (hours)

Wind speed (knots) v \ t	5	10	15	20	30	40	50
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

Problem 5. Let $f(x, y) = x^2y^3 - 2x$ and $P = (1, 1)$. Compute $D_{\mathbf{u}}f(P)$ for each unit vector $\mathbf{u}$ given below.

a. $\mathbf{u}$ in the direction of $\mathbf{v} = \langle 3, 3 \rangle$

b. $\mathbf{u}$ in the direction from P to $Q = (1, 2)$

c. $\mathbf{u}$ in the direction of maximum rate of change

d. $\mathbf{u}$ in the direction of minimum (ie. most negative) rate of change

e. $\mathbf{u}$ in the direction perpendicular to $\nabla f(P)$

Problem 6. You're exploring a landscape where the elevation z at any point is modeled by the function

$$z = 1000 - 0.005x^2 - 0.01y^2$$

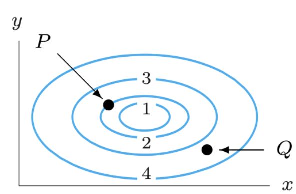
where x, y, and z are measured in meters. Suppose the positive x-direction points east and the positive y-direction points north, and suppose that you're currently standing at $(60, 40)$.

a. You're considering moving directly south from your current spot. Will you begin ascending or descending? What is the instantaneous rate of change of your elevation if you move directly south?

b. Which direction should you walk so that you'll be moving in the steepest direction? What is the instantaneous rate of change of your elevation if you move in this direction?

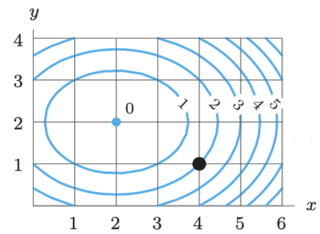
c. You're feeling tired and want to avoid climbing or descending. Your goal is to keep your elevation unchanged. Identify two different directions (give unit vectors in $\mathbb{R}^2$) you could take from your current position to maintain a constant elevation, at least for your next step.

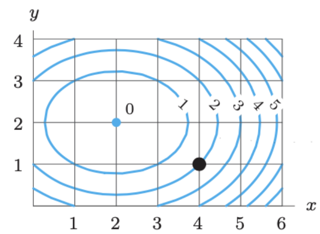
Problem 7. Consider the function f whose contour plot is shown below. Determine the sign (positive, negative, or zero) of the following quanitities.

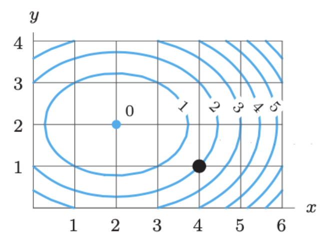


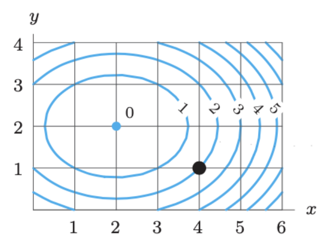
Derivative	Sign
The y-component of $\nabla f(P)$	
$D_{\mathbf{u}}f(Q)$ where $\mathbf{u} = \langle -1, 1 \rangle$	
$D_{\mathbf{u}}f(P)$ where $\mathbf{u} = \nabla f(Q)$	
$\nabla f(P) \cdot \mathbf{i}$	
$\nabla f(Q) \cdot (-\mathbf{j})$	
$\lVert \nabla f(P) \rVert - \lVert \nabla f(Q) \rVert$	

Problem 8. Consider the function f whose contour plot is shown four times below. In the first copy of the contour plot, sketch a vector with basepoint $(4,1)$ in the direction of $\nabla f(4,1)$. In the second copy, sketch a vector with basepoint $(4,1)$ in a direction $\mathbf{u}$ such that $D_{\mathbf{u}}f(4,1)=0$. In the third copy, sketch a vector with basepoint $(5,2)$ in the direction of $-\nabla f(5,2)$. In the fourth copy, sketch a vector with basepoint $(5,2)$ in the direction $\mathbf{u}$ such that $D_{\mathbf{u}}f(5,2)=0$.









Problem 9. If you liked the problems above or want more practice, our textbook has more great problems. Many odd-numbered ones have solutions in the back. Here are some that I recommend (as optional, not to be turned in):

- Section 12.4: 9-12, 19-20 (use local linear approximation)
- Section 12.6: 1-5, 7-23 odd
- Section 12.7: 17-20

Feel free to try others, including the problems in the main sections, which include full explanations.