Math 203, Spring 2026 — Homework 6

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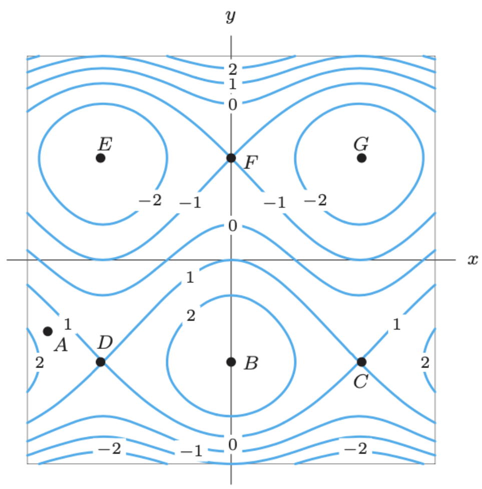
Due March 27

Instructions. This problem set contains problems from Week 7 of class. The problem numbers refer to the PDF version of our textbook, *Apex Calculus*, by Gregory Hartman.

Problem 1. Consider the contour plot below, which contains the points A, B, C, D, E, F, G. For each point, classify it as a *local minimum*, a *local maximum*, a *saddle point*, or *neither*.



Point	Classification
A	
B	
C	
D	
E	
F	
G	

Problem 2. Find the critical points of the function $f(x, y) = \frac{1}{2}x^2 + 3y^3 + 9y^2 - 3xy + 9y - 9x$ and classify them using the Second Derivative Test.

Problem 3. Find the critical points of the function $f(x, y) = x^3 + y^3 - 6y^2 - 3x + 9$ and classify them using the Second Derivative Test.

Problem 4. A closed rectangular box with a volume of 16 cubic inches is constructed using two types of materials. The top and bottom are made of material that costs 10 cents per square inch, while the four sides are made of material that costs 5 cents per square inch. Let x, y, z denote the length, width, and height of the box respectively. Write the total cost of materials to construct the box as a function of x and y only. The determine the dimensions of the box that minimize cost by finding a critical point of the function and verifying using the Second Derivative Test that the critical point you found was a local minimum. Conclude by stating the total minimal cost.

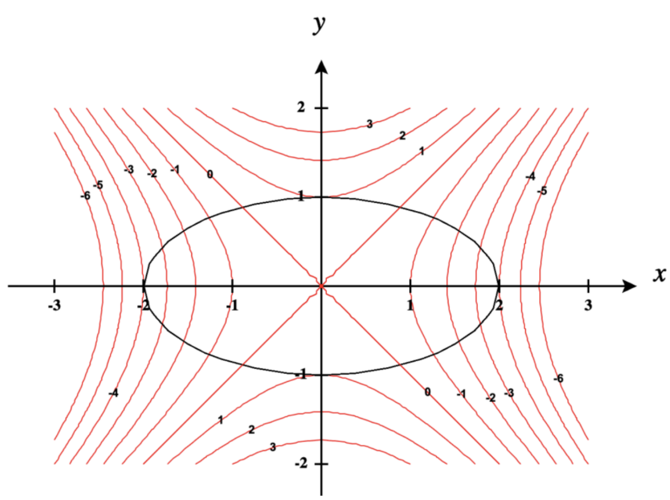
Problem 5. Let $f(x, y) = x^2 + y^2 + y + 1$. Find the global extrema of f on the closed and bounded region D whose boundary given by the triangle with vertices $(0, 1)$, $(-1, -1)$, and $(1, -1)$ and whose interior is the region enclosed by this triangle.

Problem 6. Let $f(x, y) = 5x - 7y$. Find the global extrema of f on the closed and bounded region D whose boundary is given by the curves $y = x^2$ and $y = 1$ and whose interior is enclosed by this boundary.

Problem 7. The figure below shows the contour plot of $f(x, y) = y^2 - x^2$ together with the curve $\frac{1}{4}x^2 + y^2 = 1$.

a. Use the figure to explain in a few sentences which points on the curve are where the function f attains its maximum and minimum values (subject to constraining the domain of f to be only the curve). State both the coordinates of these points and the corresponding maximum and minimum values of f.

b. Use the method of Lagrange multipliers to find the maximum and minimum values of $f(x, y)$ subject to the constraint that $\frac{1}{4}x^2 + y^2 = 1$.



Problem 8. Use the method of Lagrange multipliers to find maximum and minimum values of $f(x, y) = xy$ subject to the constraint that $4x^2 + 8y^2 = 16$.

Problem 9. A rectangular box with a square bottom and no top is to be made with 12 square meters of cardboard. That is, the box has an outer surface area of 12 square meters. Use the method of Lagrange multipliers to find the maximum volume of such a box.

Problem 10. A community relief group has raised \$2000 to provide food supplies to families in a rural region. They plan to purchase two types of food packages: sacks of grain (costing \$5 each) and sacks of lentils (costing \$10 each). Suppose the number of people who can be supported with x sacks of grain and y sacks of lentils is modeled by

$$f(x, y) = \frac{1}{2 \cdot 10^8} x^2 y^2 + x + 2y.$$

a. Write down an equation that represents the group's total spending constraint.

b. Our goal is to feed the greatest possible number of people. Use the method of Lagrange multipliers to determine the optimal allocation. Clearly state how many sacks of each type should be purchased and the maximum number of people who can be fed under this allocation.

Problem 11. In economics, the Cobb-Douglas production function is used to model the production level of a firm given levels of input like labor and capital. A company has determined that its production level is given by the Cobb-Douglas function $f(x, y) = 2.5x^{0.45}y^{0.55}$ where x represents the total number of labor hours in 1 year and y represents the total capital input for the company in 1 year. Suppose 1 unit of labor costs \$40 and 1 unit of capital costs \$50. Use the method of Lagrange multipliers to find the maximum value of f subject to the budgetary constraint of \$500,000 per year.

Problem 12. If you liked the problems above or want more practice, our textbook has more great problems. Many odd-numbered ones have solutions in the back. Here are some that I recommend (as optional, not to be turned in):

- Section 12.8: 1-3, 5-13 odd, 17, 18

Feel free to try others, including the problems in the main sections, which include full explanations.