Math 203, Spring 2026 — Homework 7

Tim Chumley

Due April 10

Instructions. This problem set contains problems from Weeks 9 and 10 of class. The problem numbers refer to the PDF version of our textbook, *Apex Calculus*, by Gregory Hartman.

Problem 1. Compute the following integrals.

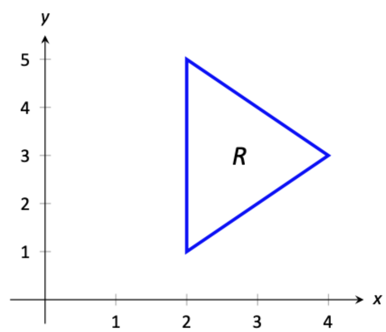
a. $\int_0^3 \int_0^4 (4x + 3y)\, dxdy$

b. $\int_0^1 \int_0^2 x^2 y\, dydx$

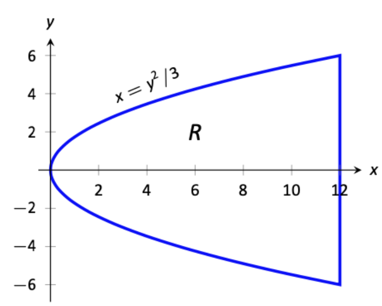
c. $\int_0^3 \int_0^y \sin x\, dxdy$

d. $\int_0^1 \int_0^1 ye^{xy}\, dxdy$

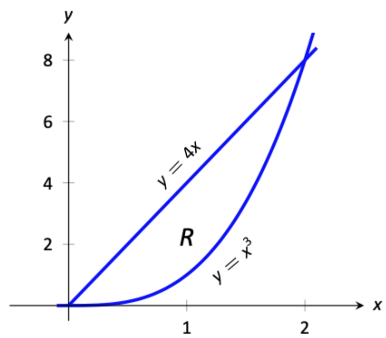
Problem 2. For each region R below, express the area of R as two different iterated integrals: $\iint_R 1\, dA$ using (1) $dA = dydx$ and (2) $dA = dxdy$. No need to evaluate the integrals (but you can and check that the value is the same with either order of integration).



a.



b.



c.

Problem 3. Each iterated integral below represents the area of a region R in the xy-plane. Give the equivalent iterated integral with the opposite order of integration. No need to evaluate the integrals (but you can and check that the value is the same with either order of integration).

a. $\int_0^1 \int_{5-5x}^{5-5x^2} dydx$

b. $\int_{-2}^2 \int_0^{4-x^2} dydx$

c. $\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} dxdy + \int_1^4 \int_{y-2}^{\sqrt{y}} dxdy$

Problem 4. For each double integral below, sketch the region R given by the problem and set up iterated integrals for both orders of integration ($dxdy$ and $dydx$). No need to evaluate the integrals (but you can use software like Wolfram Alpha to compute them and check that the value is the same with either order of integration).

a. $\iint_R ye^x \, dA$ where R is bounded by $x = 0, x = y^2, y = 1$.

b. $\iint_R (6 - 3x - 2y) \, dA$ where R is bounded by $x = 0, y = 0, 3x + 2y = 6$.

c. $\iint_R (x^3y - x) \, dA$ where R is bounded by the circle $x^2 + y^2 = 9$ in the first and second quadrants.

Problem 5. Compute each of the following integrals by reversing the order of integration.

a. $\int_0^1 \int_y^1 e^{x^2} \, dxdy$

b. $\int_0^1 \int_y^1 \sin(x^2) \, dxdy$

c. $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2 + x^3} \, dxdy$

Problem 6. Let D be the square in the xy-plane centered at the origin and given by $-1 \le x \le 1$ and $-1 \le y \le 1$. Let L be the left half of this square (ie. (x, y) points where $-1 \le x \le 0$ and $-1 \le y \le 1$), and let T be the top half of this square (ie. (x, y) points where $-1 \le x \le 1$ and $0 \le y \le 1$. Determine the sign (positive, negative, or zero) of the following double integrals.

Integral	Sign
$\iint_D x\, dA$	
$\iint_D y\, dA$	
$\iint_D (1 - x^2)\, dA$	
$\iint_D (-1 + y^2)\, dA$	
$\iint_L x\, dA$	
$\iint_L y\, dA$	
$\iint_T x\, dA$	
$\iint_T y\, dA$	

Problem 7. For description in polar coordinates below, make a sketch of the given region.

a. $1 \le r \le 2$

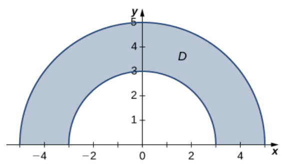
b. $\pi/4 \le \theta \le 7\pi/4$, $r \ge 0$

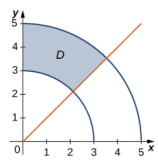
c. $\pi/4 \le \theta \le 5\pi/4$, $0 \le r \le 3$

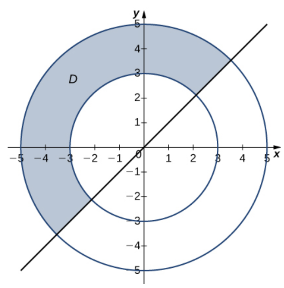
d. $-\pi/2 \le \theta \le \pi/2$, $4 \le r \le 5$

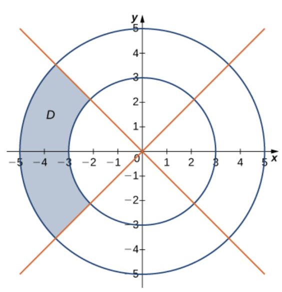
e. $-3\pi/4 \le \theta \le -\pi/4$, $0 \le r \le 1$

Problem 8. For each region in the xy-plane shown below, describe it using inequalities involving the polar variables r and θ.

a. 

b. 

c. 

d. 

Problem 9. For each double integral below, (1) sketch the region of integration, (2) and set up the integral in polar coordinates. No need to compute the integrals.

a. $\int_0^{\sqrt{2}/2}\int_{-\sqrt{1-y^2}}^{-y}(2x+y)\,dxdy$

b. $\int_{-\sqrt{2}/2}^{\sqrt{2}/2}\int_{-x}^{\sqrt{1-x^2}}(x+y)\,dydx+\int_{\sqrt{2}/2}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}(x+y)\,dydx$

c. $\int_0^1\int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} x\,dydx+\int_1^2\int_0^{\sqrt{4-x^2}} x\,dydx.$

Problem 10. For each function $f(x, y)$ and region R given below, compute $\iint_R f(x, y)\, dA$ using polar coordinates.

a. $f(x, y) = 4xy$, R is the portion of the annulus with inner and outer radiuses 1 and 2 in the third and fourth quadrants of the xy-plane.

b. $f(x, y) = 1 - x^2 - y^2$, R is the portion of the disk of radius 3 in the first, third, and fourth quadrants of the xy-plane.

c. $f(x, y) = x + y$, R is the region between $x^2 + y^2 = 4$ and $x^2 + y^2 = 16$ in the right half of the xy-plane.

Problem 11. If you liked the problems above or want more practice, our textbook has more great problems. Many odd-numbered ones have solutions in the back. Here are some that I recommend (as optional, not to be turned in):

- Section 13.1: 5, 7, 9, 11, 12, 15, 17, 19, 21
- Section 13.2: 1-4, 5-21 odd
- Section 13.3: 1-2, 3-13 odd, 14

Feel free to try others, including the problems in the main sections, which include full explanations.