Math 203, Spring 2023 — Homework 9

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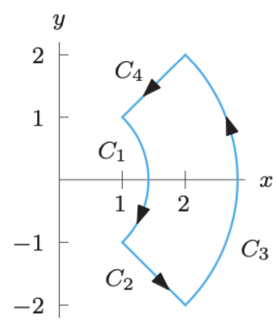
Due April 21

Instructions. This problem set has material from Week 12 of class.

Problem 1. Let $\mathbf{F}(x,y) = \langle x, y \rangle$ and consider the closed curve $C = C_1 + C_2 + C_3 + C_4$ shown below, with arrows indicating the orientation of each component C_1, C_2, C_3, C_4. Notice that C_1 and C_3 are arcs of circles centered at the origin and C_2 and C_4 are radial line segments.. Indicate the sign (positive, negative, or zero) of each of the following line integrals.



Line integral	Sign
$\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$	
$\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$	
$\int_{C_3} \mathbf{F} \cdot d\mathbf{r}$	
$\int_{C_4} \mathbf{F} \cdot d\mathbf{r}$	
$\oint_{C} \mathbf{F} \cdot d\mathbf{r}$	

Problem 2. Repeat Problem 1 using $\mathbf{F}(x,y) = \langle -y, x\rangle$.

Line integral	Sign
$\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$	
$\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$	
$\int_{C_3} \mathbf{F} \cdot d\mathbf{r}$	
$\int_{C_4} \mathbf{F} \cdot d\mathbf{r}$	
$\oint_{C} \mathbf{F} \cdot d\mathbf{r}$	

Problem 3. For each given vector field $\mathbf{F}$ and oriented curve C, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ by computing $\int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\,dt$ where $\mathbf{r}(t), a \le t \le b$, is a parametrization of C.

a. $\mathbf{F}(x,y) = \langle x, x+y\rangle$, C is the oriented curve $y = x^2$ from $(0,0)$ to $(1,1)$

b. $\mathbf{F}(x,y) = \langle xy, x\rangle$, C is the oriented curve $y = x^3$ from $(-1,-1)$ to $(1,1)$

Problem 4. For each given vector field $\mathbf{F}$ and oriented curve C, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ by computing $\int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t)\,dt$ where $\mathbf{r}(t), a \le t \le b$, is a parametrization of C.

a. $\mathbf{F}(x,y) = \langle 1, xy\rangle$, C is the right half of the unit circle oriented from $(0,-1)$ to $(0,1)$

b. $\mathbf{F}(x,y) = \left\langle x-y, x^2\right\rangle$, C consists of two line segments, oriented from $(-1,0)$ to $(-1,1)$ to $(4,1)$

Problem 5. For each given vector field $\mathbf{F}$, show that it's conservative by finding a potential function f and use the Fundamental Theorem of Line Integrals to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is any piecewise smooth curve from $(1,2)$ to $(3,4)$.

a. $\mathbf{F}(x,y) = \langle y+1, x\rangle$

b. $\mathbf{F}(x,y) = \langle 2x+y, 2y+x\rangle$

Problem 6. For each given vector field $\mathbf{F}$, show that it's conservative by finding a potential function f and use the Fundamental Theorem of Line Integrals to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is any piecewise smooth curve from $(1,2)$ to $(3,4)$.

a. $\mathbf{F}(x,y) = \langle 2x, 2y\rangle$

b. $\mathbf{F}(x,y) = \left\langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}\right\rangle$

Problem 7. For each given vector field $\mathbf{F}$ and piecewise smooth, closed, positively oriented curve C, compute $\operatorname{curl}\mathbf{F}$ and use Green's Theorem to compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

a. $\mathbf{F}(x,y) = \langle x-y, x+y\rangle$, C is the parabola $y = x^2$ from $(0,0)$ to $(2,4)$ along with the line segment from $(2,4)$ to $(0,0)$.

b. $\mathbf{F}(x,y) = \langle -y, x\rangle$, C is the unit circle

Problem 8. For each given vector field $\mathbf{F}$ and piecewise smooth, closed, positively oriented curve C, compute $\operatorname{curl}\mathbf{F}$ and use Green's Theorem to compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

a. $\mathbf{F}(x,y) = \left\langle 0, x^2\right\rangle$, C is the triangle with vertices $(0,0), (2,0)$, and $(1,1)$

b. $\mathbf{F}(x,y) = \langle x+y, 2x\rangle$, C is the piecewise smooth curve bounding the region between $y = 1 - x^2$ and $y = x - 1$.