12.3 Partial Derivatives

Def Let $f(x,y)$ be a given function.

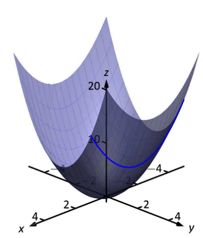
The partial derivative of f with respect to x, denoted f_x or $\frac{\partial f}{\partial x}$, is the derivative of f where we treat y like a constant.

The partial derivative of f with respect to y, denoted f_y or $\frac{\partial f}{\partial y}$, is the derivative of f where we treat x like a constant.

Example Let $f(x,y) = x^2 + 2y^2$

① Compute f_x, $f_x(-2,2)$, $f_x(0,2)$, $f_x(2,2)$ and discuss what this means geometrically about slopes along the curve $z = f(x,2)$.

$f_x = 2x$



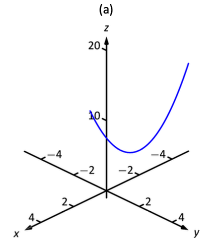
(a)

$f_x(-2,2) = -4 \Rightarrow$ negative slope at $(-2,2)$

$f_x(0,2) = 0 \Rightarrow 0$ slope at $(0,2)$

$f_x(2,2) = 4 \Rightarrow$ positive slope at $(2,2)$

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{slopes along curve where } y=2}$



(b)

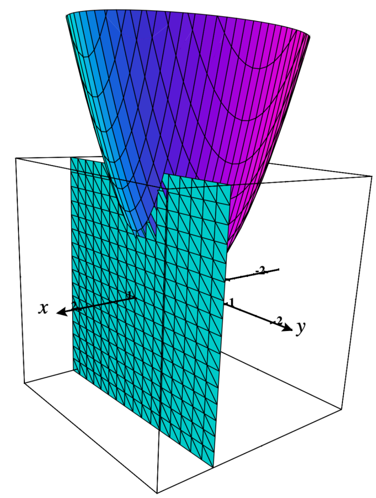
② Do the same with f_y, $f_y(1,-2)$, $f_y(1,0)$, $f_y(1,2)$

$f_y = 4y$

$f_y(1,-2) = -8 \Rightarrow$ negative slope at $(1,-2)$

$f_y(1,0) = 0 \Rightarrow$ 0 slope at $(1,0)$

$f_y(1,2) = 8 \Rightarrow \underbrace{\text{positive slope at } (1,2)}_{\text{slopes along curve where } x=1}$



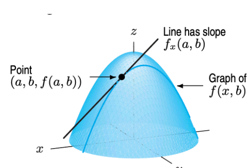
General Rules

① $f_x(a,b)$ gives the slope at $(a,b,f(a,b))$ of the curve formed by intersecting graph of $f(x,y)$ and the plane $y=b$.

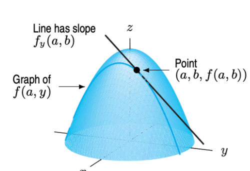
describes Δz as x increases with y fixed

② $f_y(a,b)$ gives the slope at $(a,b,f(a,b))$ of the curve formed by intersecting graph of $f(x,y)$ with $x=a$.

describes Δz as y increases with x fixed

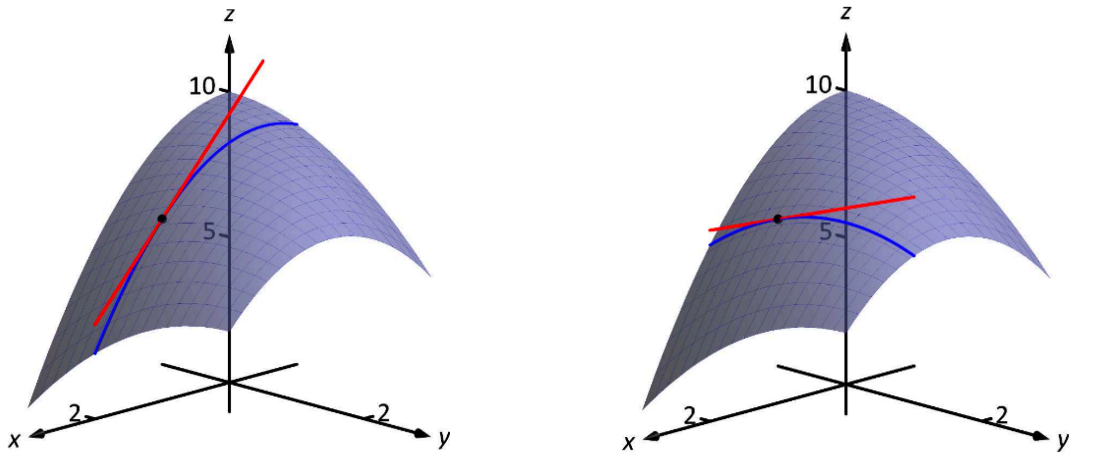


negative slope



negative slope

Example Let $f(x,y) = -x^2 - \frac{1}{2}y^2 + xy + 10$. Find $f_x(2,1)$ and $f_y(2,1)$ and interpret their values geometrically.



(a) (b)

$f_x = -2x + y$,

$f_x(2,1) = -2(2) + 1 = -3 < 0$

(negative slope)

$f_y = -y + x$,

$f_y(2,1) = -1 + 2 = 1 > 0$

(positive slope)

Problem 1. Compute f_x and f_y for each function below.

a. $f(x,y) = 5x^2y^3 + 8xy^2 - 3x^2$

b. $f(x,y) = \sin(x^2y^3) + \cos(y^2)$

c. $f(x,y) = xe^{x^2y^2}$

(a) $f_x = 10xy^3 + 8y^2 - 6x$

$f_y = 15x^2y^2 + 16xy$

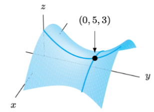
(b) $f_x = 2xy^3\cos(x^2y^3)$

$f_y = 3x^2y^2\cos(x^2y^3) - 2y\sin(y^2)$

(c) $f_x = e^{x^2y^2} + 2x^2y^2e^{x^2y^2}$ (using product rule)

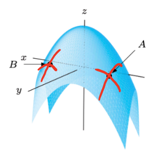
$f_y = 2x^3y\,e^{x^2y^2}$

Problem 2. The figure below shows the graph of $f(x, y)$. What are the signs of $f_x(0, 5)$ and $f_y(0, 5)$?



$f_x(0,5)$ negative, $f_y(0,5)$ positive

Problem 3. The figure below shows the graph of $f(x, y)$. What are the signs of $f_x(A)$, $f_y(A)$, $f_x(B)$, and $f_y(B)$?



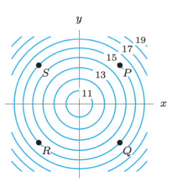
$f_x(A)$ positive

$f_x(B)$ negative

$f_y(A)$ negative

$f_y(B)$ negative

Problem 4. The figure below shows a contour plot of $f(x, y)$. Notice the curves are labeled with corresponding z values. What are the signs of f_x and f_y at the points P, Q, R, and S?



$f_x(P) > 0$ $f_x(Q) > 0$ $f_x(R) < 0$ $f_x(S) < 0$

$f_y(P) > 0$ $f_y(Q) < 0$ $f_y(R) < 0$ $f_y(S) > 0$