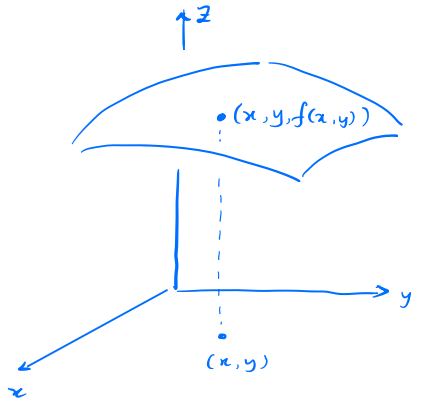
12.1 Intro to Multivariable Functions

Def The graph of a function $f(x,y)$ of two variables is the set of points (x,y,z) where $z = f(x,y)$.



Graphs are surfaces in $\mathbb{R}^3$

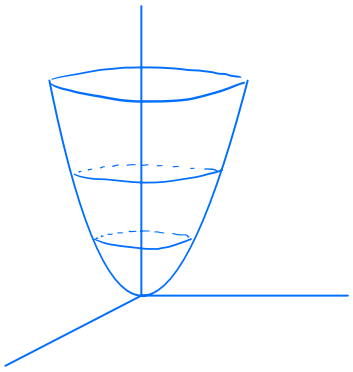
The domain of such a function is a subset D of $\mathbb{R}^2$ (2 inputs) and its range is a subset of $\mathbb{R}$ (1 output)

Example $f(x,y)$ = elevation at coordinate (x,y) in mountainous region, graph is 3d surface plot of region

Example $f(x,y) = x^2 + y^2$

Domain $= \mathbb{R}^2$ Range $= [0, \infty)$ (non-negative outputs)

Plot :

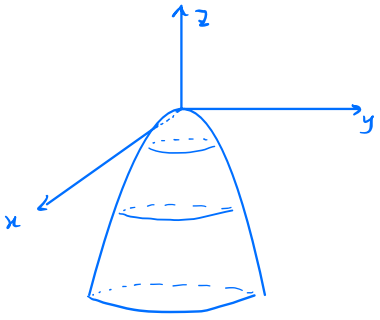


graph of $f(x,y) = x^2 + y^2$

called a paraboloid.

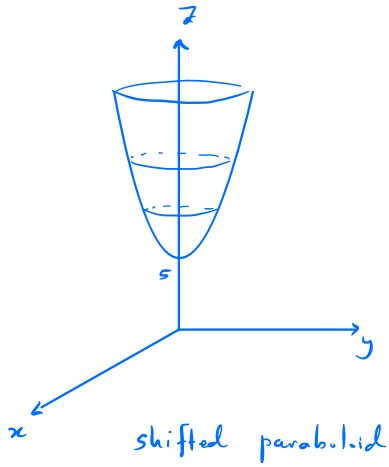
Example Plot the graphs of $g(x,y) = -x^2 - y^2$

and $h(x,y) = 5 + x^2 + y^2$.



upside down paraboloid

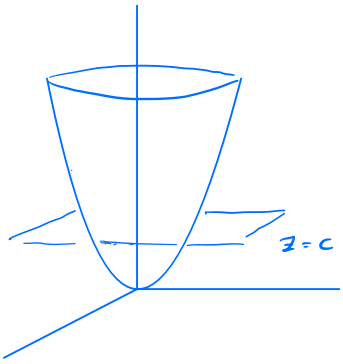
range $= (-\infty, 0]$



shifted paraboloid.

range $= [5, \infty)$

Question What is the intersection between the plane $z = c$ and the graph of $f(x,y) = x^2 + y^2$?

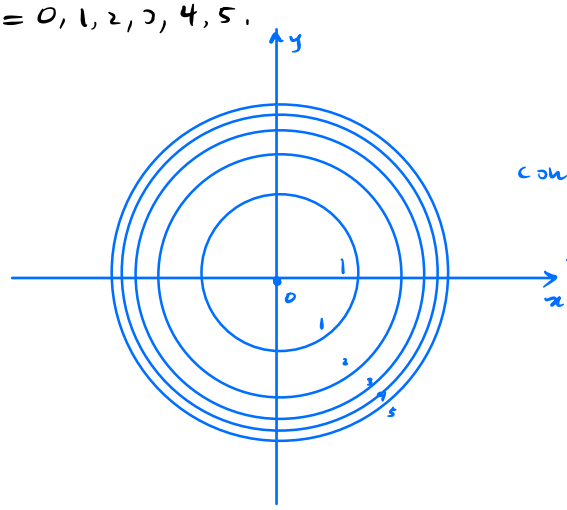


$$\begin{cases} z = c \\ z = x^2 + y^2 \end{cases} \Rightarrow x^2 + y^2 = c, \text{ a circle of radius } \sqrt{c}$$

Def The **level curves** of a function $f(x,y)$ are curves in $\mathbb{R}^2$ (the xy-plane) of the form $c = f(x,y)$. A **contour plot** is a plot in xy-plane that contains multiple level curves.

Example Make contour plot of $f(x,y) = x^2 + y^2$ using $c = 0, 1, 2, 3, 4, 5$.

c	radius
0	0
1	1
2	$\sqrt{2} \approx 1.4$
3	$\sqrt{3} \approx 1.73$
4	2
5	$\sqrt{5} \approx 2.23$

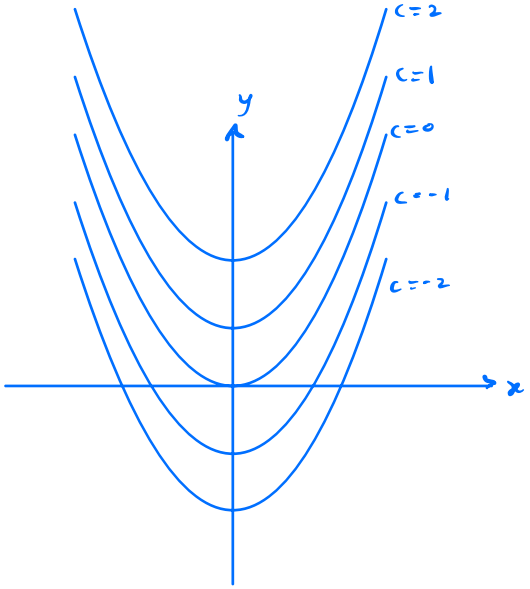


contour plot is like a topographical map showing elevations

Example Make a contour plot of $f(x,y) = y - x^2$

for $c = -2, -1, 0, 1, 2$. Notice the level curves are of the form $c = y - x^2$ or $y = x^2 + c$.

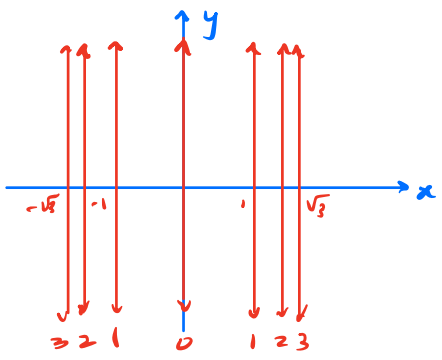
c	equation
-2	$y = x^2 - 2$
-1	$y = x^2 - 1$
0	$y = x^2$
1	$y = x^2 + 1$
2	$y = x^2 + 2$

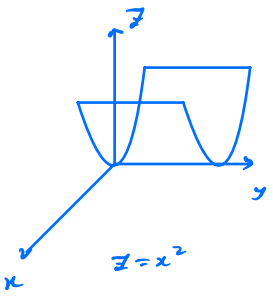


Example Make a contour plot of $f(x,y) = x^2$

for $c = 0, 1, 2, 3$. Here level curves are of the form $x^2 = c$ or $x = \pm\sqrt{c}$, which are pairs of vertical lines.

c	lines
0	$x = 0$
1	$x = \pm 1$
2	$x = \pm\sqrt{2}$
3	$x = \pm\sqrt{3}$





Problem 1. Consider the function $f(x,y) = \sqrt{x^2+y^2}$.

a. What are the domain and range of f?

b. Make a contour plot of the level curves of f using $c = 0, 1, 2, 3, 4, 5$.

c. What is different about this contour plot compared to that of $g(x,y) = x^2 + y^2$?

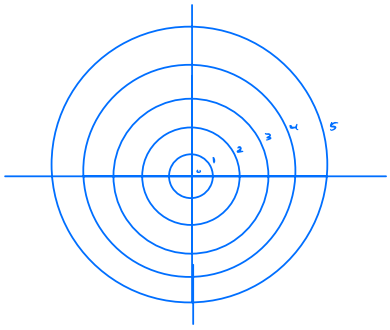
d. Use CalcPlot3D to make a sketch of the graph of f.

e. Sketch the graphs of $h(x,y) = -\sqrt{x^2+y^2}$ and $k(x,y) = 1 - \sqrt{x^2+y^2}$.

(a) domain $= \mathbb{R}^2$, range $= [0, \infty)$

(b) level curves: $c = \sqrt{x^2+y^2}$

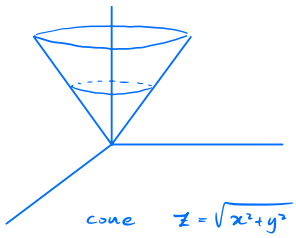
$c^2 = x^2 + y^2$, circles of radius c



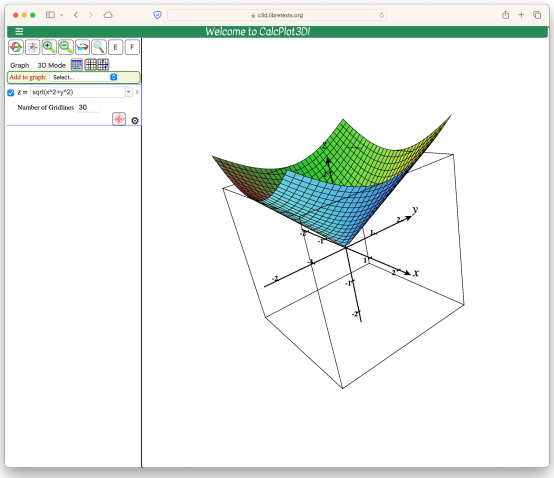
(c) the circles have radius c instead of $\sqrt{c}$

So they're evenly spaced as c increases

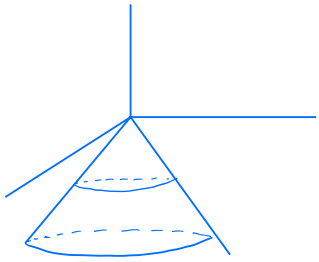
(d)



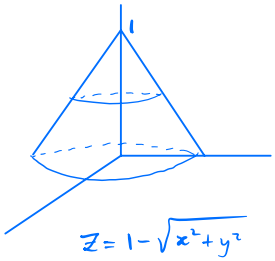
cone $z = \sqrt{x^2+y^2}$



(e)



$z = -\sqrt{x^2+y^2}$



$z = 1 - \sqrt{x^2+y^2}$

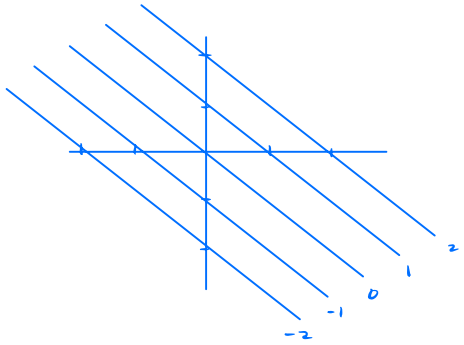
Problem 2. Consider the function $f(x, y) = x + y$.

a. Make a contour plot using level curves $c = -2, -1, 0, 1, 2$.

b. What do you believe the graph of f looks like?

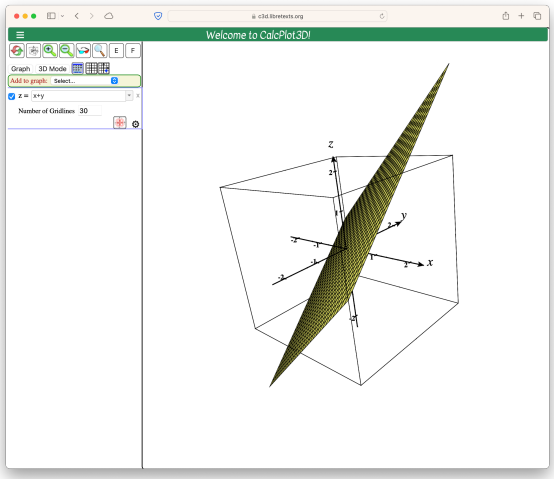
c. Use CalcPlot3D to make a sketch of the graph of f.

(a) $x + y = c$ is a line with x- and y-intercepts both c

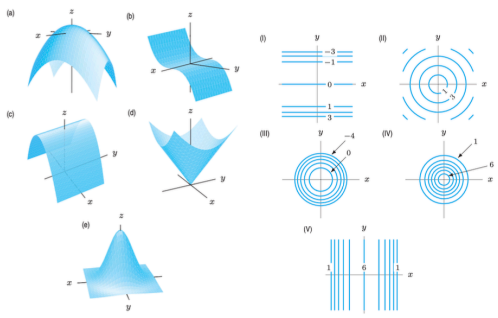


(b) plane

(c)



Problem 3. Match the surfaces (a)-(e) with their corresponding contour diagrams (I)-(V) shown below.



Surface	Contour plot
a	III
b	I
c	V
d	II
e	IV