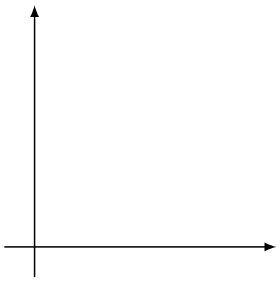
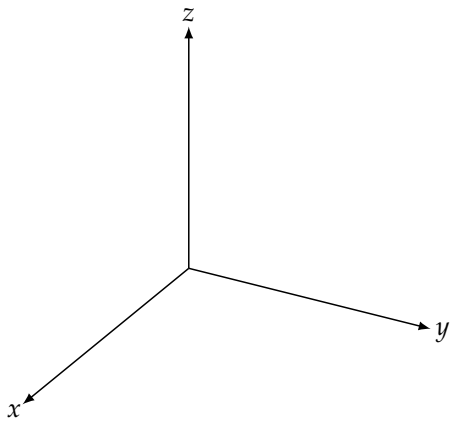
Let's practice setting up triple integrals using different orders of integration from $dV = dz\,dy\,dx$.

Example 1. Consider the iterated integral

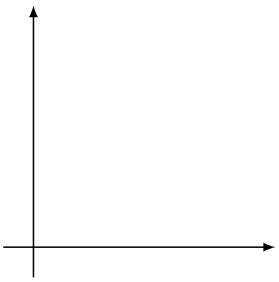
$$\int_0^1 \int_0^{2x} \int_0^{1-\frac{y}{2}} f(x,y,z)\,dz\;dy\,dx.$$

(a) Sketch the projection R of the region of integration D onto the xy-plane, and sketch D.

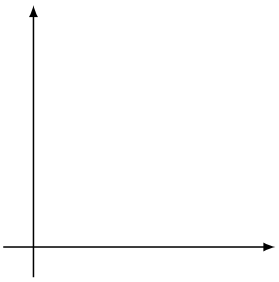




(b) Set up the triple integral using $dV = dy\,dz\,dx$.

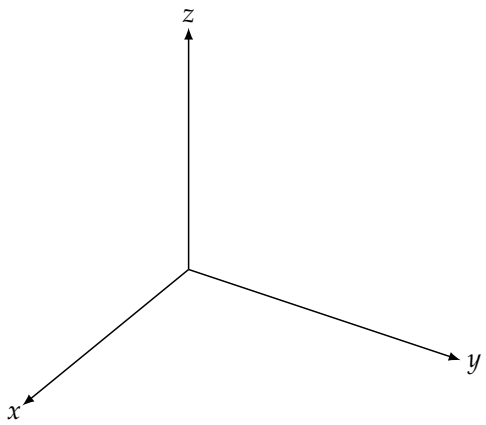


(c) Set up the triple integral using $dV = dx\,dz\,dy$.

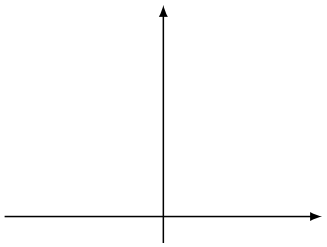


Example 2. Consider the solid region D bounded by the surfaces $y = x^2$, $z = 2 - \frac{y}{2}$, and $z = 0$.

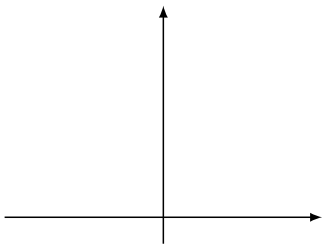
Draw and label the solid region D. Set up a triple integral for the volume of D using the following choices of dV.



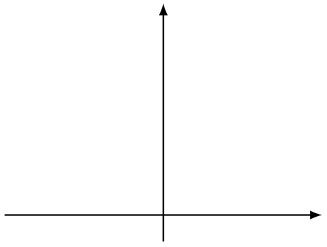
(a) $dV = dz\,dy\,dx$



(b) $dV = dy\,dz\,dx$



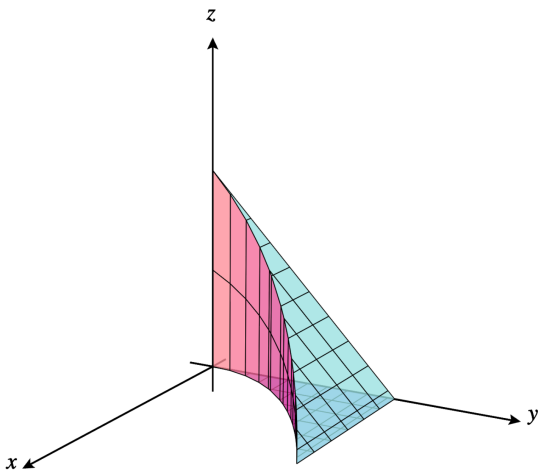
(c) $dV = dx\,dy\,dz$



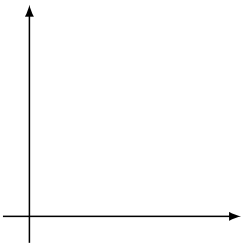
Example 3. The figure shows the region of integration D for the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\, dz\, dy\, dx.$$

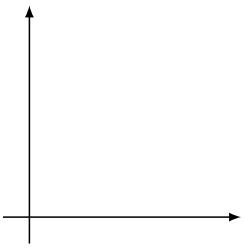
Label the surfaces that bound the solid region D. Rewrite this integral using:



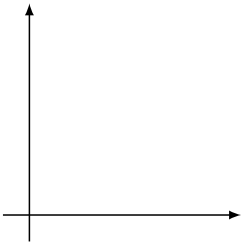
(a) $dV = dz\, dx\, dy$



(b) $dV = dx\, dz\, dy$



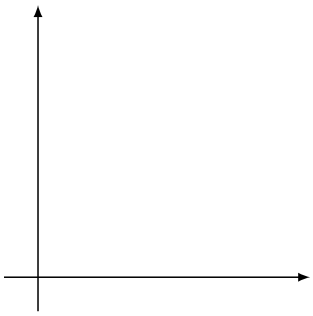
(c) $dV = dy\, dz\, dx$ (caution: kind of tricky!)

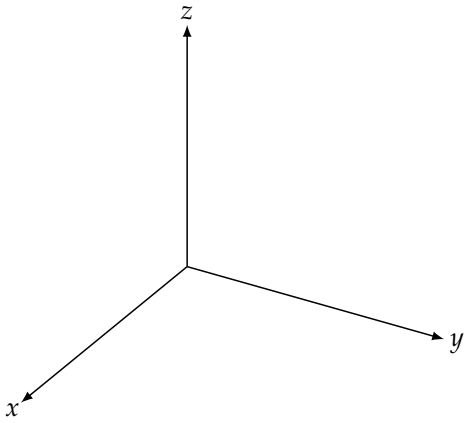


Example 4. Consider the iterated integral

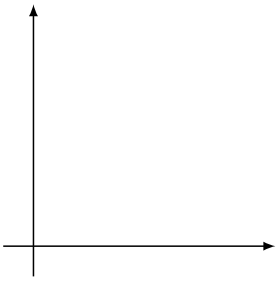
$$\int_0^1 \int_0^{2-x} \int_0^3 f(x,y,z)\,dz\;dy\,dx.$$

(a) Sketch the projection R of the region of integration D onto the appropriate coordinate plane, and sketch D.

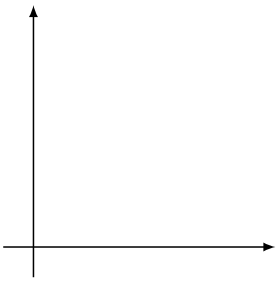




(b) Set up the triple integral using $dV = dy\,dz\,dx$.



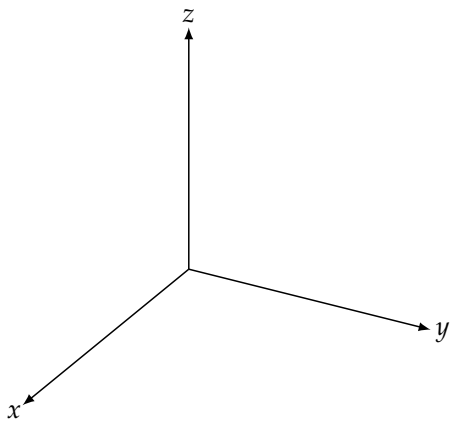
(c) Set up the triple integral using $dV = dx\,dz\,dy$. (*Hint*: we'll need to split it into a sum of two integrals.)



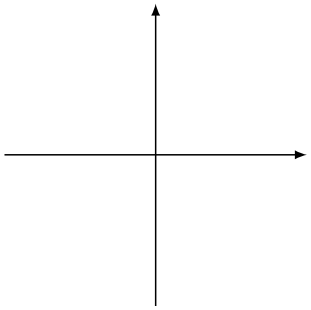
More Practice with Triple Integrals

Example 5. Let D be the solid region bounded between the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 3$.

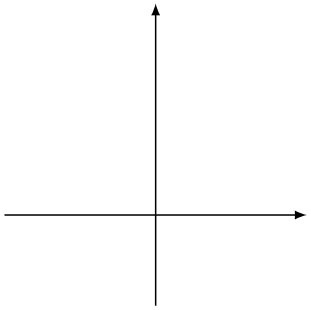
(a) Sketch and label D.



(b) Set up the triple integral using $dV = dz\,dy\,dx$.



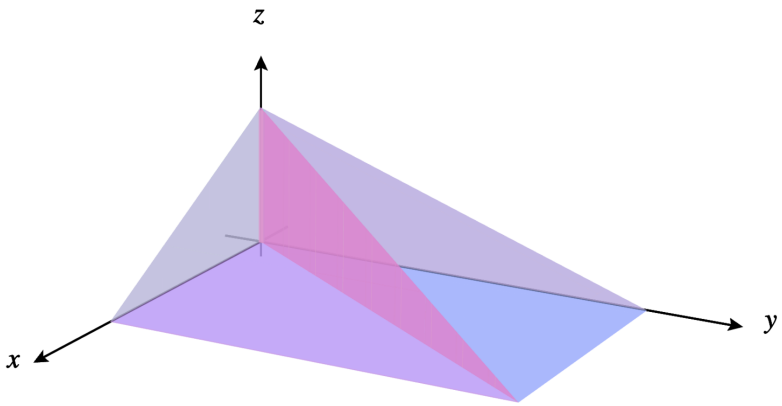
(c) Set up the triple integral using $dV = dx\,dy\,dz$.



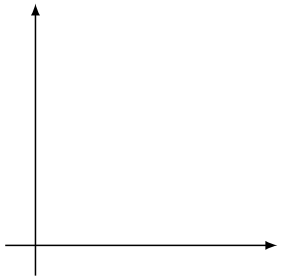
Example 6. The figure shows the region of integration D——a pyramid-shaped solid with a rectangular base——for the integral

$$\int_0^4 \int_0^{1-\frac{y}{4}} \int_0^{2-2z} f(x,y,z)\,dx\,dz\,dy.$$

Label the surfaces that bound the D. Rewrite this integral using:



(a) $dV = dy\,dz\,dx$



(b) $dV = dz\,dy\,dx$ (*Hint*: we'll need to split it into a sum of two integrals.)