Math 203 — Critical Points and the Second Derivative Test

Problem 1. True or false?

a. If $f_x(a,b) = 0$ and $f_y(a,b) = 0$, then (a,b) is a critical point of f.

b. The point $(0,0)$ is a critical point of $f(x,y) = \sqrt{x^2+y^2}$.

c. If (a,b) is a critical point of f, then $\nabla f(a,b) = \mathbf{0}$.

d. If f_x and f_y exist at (a,b) and f has a local extremum (ie. local minimum or local maximum) at (a,b) then the tangent plane of f at (a,b) is horizontal (ie. parallel to the xy-plane).

e. If the tangent plane of f at (a,b) is horizontal, then f has a local extremum at (a,b).

Problem 2. Find all critical points of the following functions and use the Second Derivative Test to classify them.

a. $f(x,y) = \frac{1}{2}x^2 + 2y^2 - 8y + 4x$

b. $f(x,y) = 4 + x^3 + y^3 - 3xy$

c. $f(x,y) = x^2 + y^3 - 3y + 1$

Problem 3. Use CalcPlot3d to visualize the graphs and contour plots of the functions in Problem 2 and informally verify that your classifications of critical points are accurate.