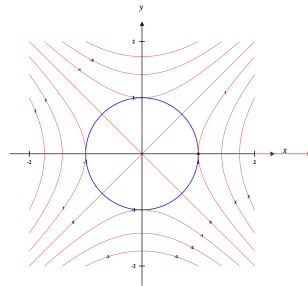


Math 203 — Lagrange Multipliers

Problem 1. Find the global extrema of $f(x, y) = x^2 - y^2$ subject to the constraint $x^2 + y^2 = 1$. You might find the contour diagram below helpful in checking your answer.



Problem 2. Find the global extrema of $f(x, y) = x^2 + y^2 + 2x - 2y + 1$ subject to the constraint $x^2 + y^2 = 2$.

Problem 3. A manufacturer of golf balls has created a model (ie. a function)

$$f(x, y) = 48x + 96y - x^2 - 2xy - 9y^2$$

which outputs the profit of monthly sales (in thousands of dollars), given x golf balls sold per month (in thousands) and y hours per month of advertising. Every thousand golf balls cost \$20 (thousand dollars) to produce and every hour of advertising costs \$4 (thousand dollars). Find values of x and y that maximize profit subject to the constraint that there is a fixed budget of \$216 thousand dollars.

Problem 4. An economic model for consumer behavior tries to capture how much happiness (or *utility*) an individual derives from combinations of leisure and goods. In this model, utility U is given as a function $f(x, y)$ of the amount of leisure x and goods y the individual has. We suppose

$$U = f(x, y) = x^{1/3}y^{2/3}$$

and we suppose that x and y are non-negative quantities. Since life has constraints, it is natural to assume that the amount of leisure and goods an individual can have are constrained by the equation

$$3x + 2y = 12.$$

Find the optimal combination of leisure and goods that maximizes utility.

