Math 203 — More double integrals

Problem 1. Let D be the region inside the unit circle in $\mathbb{R}^2$, let R be the right half of D, and let B be the bottom half of D. Determine whether each of the following double integrals is positive, negative, or zero by thinking about the (signed) volume that each represents and whether the function in the integrand is positive, negative, or zero over the region of integration. You might find it helpful to use CalcPlot3d.

a. $\iint_D 1 \, dA$

b. $\iint_D (1 - \sqrt{x^2 + y^2}) \, dA$

c. $\iint_D (-1 + x^2 + y^2) \, dA$

d. $\iint_R x \, dA$

e. $\iint_R y \, dA$

f. $\iint_B x \, dA$

g. $\iint_B y \, dA$

Problem 2. Each double integral below cannot be computed using the given order of integration. Sketch the region of integration, set up the integral again with the reversed order of integration, and compute its value.

a. $\int_0^2 \int_{y/2}^1 e^{x^2} \, dxdy$

b. $\int_0^1 \int_{\sqrt{y}}^1 e^{-x^3} \, dxdy$

c. $\int_{-1}^0 \int_{-y}^1 \sin(x^2) \, dxdy + \int_0^1 \int_y^1 \sin(x^2) \, dxdy$

Problem 3. For each double integral below, sketch the region of integration and set up the integral with the reversed order of integration.

a. $\int_0^1 \int_{-\sqrt{1-y}}^{\sqrt{1-y}} f(x, y) \, dxdy$

b. $\int_0^1 \int_{y^2}^{(y+1)/2} f(x, y) \, dxdy$

c. $\int_{-1}^1 \int_{-1+y^2}^{\sqrt{1-y^2}} f(x, y) \, dxdy$