Math 203 — Polar integrals

Problem 1. Set up both polar and Cartesian integrals for $\iint_R f(x,y)\,dA$ where f and R are given as follows.

a. $f(x,y) = 3 + 2x - 4y$, R is the disk of radius 4.

b. $f(x,y) = (x^2+y^2)^{-5/2}$, R is the left half the annulus with inner and outer radiuses 2 and 5.

c. $f(x,y) = e^{-(x^2+y^2)}$, R is the portion of the disk of radius 6 in the second and fourth quadrants.

Problem 2. Rewrite the following integrals in polar coordinates:

a. $\int_0^{\sqrt{2}/2} \int_y^{\sqrt{1-y^2}} (x+y)\,dxdy$

b. $\int_{-\sqrt{2}/2}^0 \int_{-x}^{\sqrt{1-x^2}} (x^2+y^2)\,dydx$

c. $\int_{-1}^{-\sqrt{2}/2} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3x-y)\,dydx + \int_{-\sqrt{2}/2}^0 \int_x^{\sqrt{1-x^2}} (3x-y)\,dydx$

Problem 3. Find the volume of the solid, shown below, bounded by paraboloid $z = 1 - x^2 - y^2$ and the xy-plane.