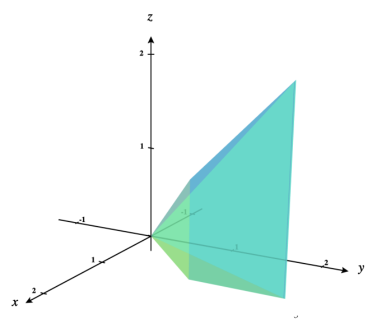
Math 203 — Triple integrals

Problem 1. Consider the triple integral $\int_0^1 \int_x^{2x} \int_0^y 2xyz\,dzdydx$. This is a triple integral over the region D shown below.

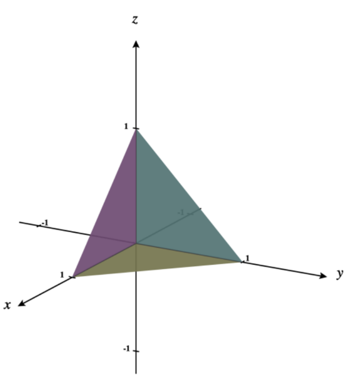


a. Identify the lower and upper bounding surfaces of D.

b. Sketch the region R given by projecting D onto the xy-plane.

c. Compute the integral.

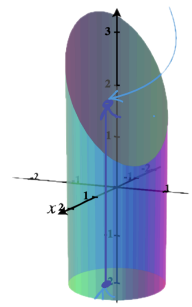
Problem 2. Let D be the solid region that is bounded by the planes $x = 0, y = 0, z = 0$, and $x + y + z = 1$. This shape is like a pyramid whose faces are all triangles. It is shown below.



a. Make a sketch in the xy-plane of the projection R of D onto the xy-plane.

b. Set up a triple integral to find the mass of D given that it has density $f(x, y, z)$.

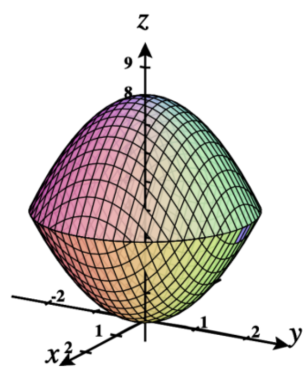
Problem 3. Let D be the solid region that is given by a solid cylinder bounded on the sides by $x^2 + y^2 = 1$ whose bottom face is the plane $z = -2$ and whose top face is the plane $x + y + z = 2$. It is shown below.



a. Make a sketch in the xy-plane of the projection R of D onto the xy-plane.

b. Set up a triple integral to find the mass of D given that it has density $f(x, y, z)$.

Problem 4. Let D be the solid region that is bounded below by the paraboloid $z = x^2 + y^2$ and above by the parabolid $z = 8 - (x^2 + y^2)$. It is shown below.



a. Find where the two paraboloids intersect.

b. Make a sketch in the xy-plane of the projection R of D onto the xy-plane.

c. Set up a triple integral to find the mass of D given that it has density $f(x, y, z)$.