Math 203 — Cylindrical coordinates

Problem 1. Set up a triple integral in cylindrical coordinates for the volume of each solid region described below. Begin by sketching the region.

a. Bounded by the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 25$ and the planes $z = 0$ and $z = 6$.

b. Bounded by the half-cylinder $x^2 + y^2 = 9$ where $x \leq 0$ with height 4 and base on the xy-plane.

c. Bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by unit sphere.

d. Bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the plane $z = 2$.

e. The solid cylinder bounded by $x^2 + y^2 = 1$ and $z = 0, z = 1$ but with the solid cone bounded by $z = \sqrt{x^2 + y^2}$ and $z = 1$ removed.

Problem 2. A triple integral in cylindrical coordinates is given. Describe the region of integration in words and with a sketch.

a. $\int_0^{\pi/2} \int_0^2 \int_0^2 r \, dz dr d\theta$

b. $\int_0^{\pi} \int_0^1 \int_0^{1-r} r \, dz dr d\theta$

c. $\int_0^{2\pi} \int_0^1 \int_0^{2-r} r \, dz dr d\theta$

Problem 3. A solid is bounded from above by the paraboloid $z = 1 - x^2 - y^2$ and from below by the xy-plane. It has density $f(x, y, z) = 15 - 3z$, meaning it is heavier near the base and lighter near the top. Use cylindrical coordinates to find the mass of the solid.