Math 203 — Exam 2 review

Your exam in class on April 3 will contain about 7 multi-part problems. It will cover material from Homework 4 to Homework 6. In the textbook, this is material spanning sections 12.1-12.8, though not any sections we skipped. Also, the Method of Lagrange Multipliers will be part of the exam. The problems below give you a sampling of some similar problems, but it's not necessarily comprehensive, so make sure to review old homework, quizzes, worksheets, and lecture notes. There are also problems in our textbook, with answers to odd-numbered problems in the back. No notes will be allowed on the exam, but there will be some formulas given on the exam sheet (eg. the discriminant D in the second derivative test) as well as the unit circle with corresponding values of sine and cosine. You'll be allowed to use a scientific calculator with no graphing or calculus functionality but the problems will be written so that a calculator is not required.

Problem 1. Consider the function $f(x, y) = 4 - (x-1)^2 - (y-2)^2$.

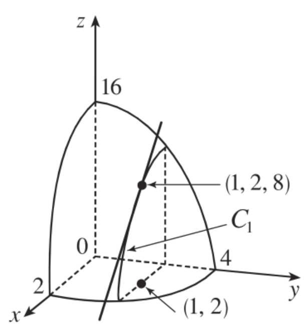
a. What are the domain and range of f?

b. Make a contour diagram using the level curves $f(x, y) = c$ where $c = 0, 1, 2, 3, 4$.

c. Explain why a contour diagram of f cannot have level curves where $c > 4$.

d. Sketch the graph of f.

Problem 2. Consider the function f whose graph is shown below.



a. Find the value of $f(1, 2)$.

b. Which of the following is an equation for the curve C_1?

1. $z = f(1, 2)$
2. $z = f(x, 2)$
3. $z = f(1, y)$

c. Find the sign (positive, negative, or zero) of the following quantities:

1. $f_x(1, 2)$
2. $f_y(1, 2)$
3. $f_{xx}(1, 2)$
4. $f_{yy}(1, 2)$
5. $D_{\mathbf{u}}f(1, 2)$ where $\mathbf{u} = \langle -1, -1 \rangle$
6. $\|\nabla f(1, 2)\| - D_{-\mathbf{i}}f(1, 2)$

Problem 3. For each of the following examples, find $D_{\mathbf{u}}f(P)$.

a. $f(x, y) = xy$, $P = (0, -2)$

   1. $\mathbf{u}$ in the direction of $\langle 1, 3\rangle$
   2. $\mathbf{u}$ in the direction of maximum rate of increase

b. $f(x, y) = e^x \sin y$, $P = (1, \pi/2)$

   1. $\mathbf{u}$ in the direction of $\langle -1, 1\rangle$
   2. $\mathbf{u}$ in the direction of maximum rate of decrease

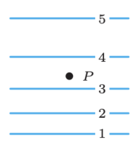
c. $f(x, y) = xe^{-y}$ $P = (1, 0)$

   1. $\mathbf{u}$ in the direction of $\langle 0, 1\rangle$
   2. $\mathbf{u}$ in the direction given by rotating ∇f by $\pi/2$ counterclockwise

d. $f(x, y) = \sqrt{x^2 + 2y}$ at $(4, 10)$

   1. $\mathbf{u}$ in the direction of $\langle 2, 0\rangle$
   2. $\mathbf{u}$ in the direction given by rotating ∇f by $\pi/2$ clockwise

Problem 4. The figure below shows a contour plot of $f(x, y)$. Use the contour plot to determine the sign (positive, negative, or zero) of $f_x(P), f_y(P), f_{xx}(P), f_{yy}(P), f_{xy}(P)$.



Problem 5. The table below gives values of the function $V = f(T, P)$ where V is the volume (in cubic feet) of one pound of steam at a temperature T (in degrees Fahrenheit) and pressure P (in pounds per square inch).

		Pressure P (lb/in^2)			
		20	22	24	26
Temperature T (°F)	480	27.85	25.31	23.19	21.39
	500	28.46	25.86	23.69	21.86
	520	29.06	26.41	24.20	22.33
	540	29.66	26.95	24.70	22.79

a. Give the local linear approximation $L(T, P)$ of V near $T = 500$ and $P = 24$. Keep in mind the following question when approximating partial derivatives.

b. Estimate the volume of a pound of steam at temperature 505 degrees Fahrenheit and pressure 24.3 pounds per square inch.

Problem 6. Consider the following functions. Find their critical points and use the Second Derivative Test to classify them.

a. $f(x, y) = y^2 + xy + 3y + 2x + 3$

b. $f(x, y) = x^2 + 2y^2 - x^2 y$

Problem 7. Find the global minimum and maximum value, as well as the location of where they occur, of the function $f(x, y) = 3y - 2x$ constrained to the region bounded by $y = x^2 - 2$ and $y = x$.

Problem 8. A food manufacturing company is planning to make cans in the shape of circular cylinders with a volume of 10 cubic centimeters. In order to save on the cost of labels (on sides as well as tops and bottoms), they would like to minimize the surface area of the can. Use the method of Lagrange multipliers to find dimensions (radius x and height y) that fit these requirements.

Problem 9. Let $f(x, y) = x^2 + 4y^2 - 2x + 8y$. Use the method of Lagrange Multipliers to determine the extreme values of f under the constraint that $x + 2y = 7$ and $x, y \geq 0$.