Math 203 — Exam 2 review

Your exam in class on March 31 will contain about 7 multi-part problems. It will cover material from Homework 4 to Homework 6 (in the textbook, this is material spanning sections 12.2-13.2, though not any sections we skipped, as well as 9.4 and the method of Lagrange multipliers). The problems below give you a sampling of some similar problems, but it's not necessarily comprehensive, so make sure to review old homework, quizzes, worksheets, and lecture notes. There are also problems in our textbook, with answers to odd-numbered problems in the back. No notes will be allowed on the exam, but there will be some formulas given on the exam sheet (eg. the discriminant D in the second derivative test) as well as the unit circle with corresponding values of sine and cosine. You'll be allowed to use a scientific calculator with no graphing or calculus functionality but the problems will be written so that a calculator is not required.

Problem 1. For each of the following examples, find $D_{\mathbf{u}}f(P)$.

a. $f(x, y) = xy$, $P = (0, -2)$

1. $\mathbf{u}$ in the direction of $\langle 1, 3\rangle$
2. $\mathbf{u}$ in the direction of maximum rate of change

b. $f(x, y) = e^x \sin y$, $P = (1, \pi/2)$

1. $\mathbf{u}$ in the direction of $\langle -1, 1\rangle$
2. $\mathbf{u}$ in the direction of minimum rate of change

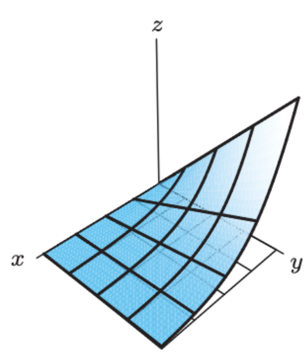
c. $f(x, y) = xe^{-y}$ $P = (1, 0)$

1. $\mathbf{u}$ in the direction of $\langle 0, 1\rangle$
2. $\mathbf{u}$ in the direction given by rotating ∇f by $\pi/2$ counterclockwise

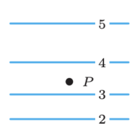
d. $f(x, y) = \sqrt{x^2 + 2y}$ at $(4, 10)$

1. $\mathbf{u}$ in the direction of $\langle 2, 0\rangle$
2. $\mathbf{u}$ in the direction given by rotating ∇f by $\pi/2$ clockwise

Problem 2. The figure below shows the graph of $f(x, y)$ on the domain $0 \leq x \leq 4$ and $0 \leq y \leq 4$. Use the graph to rank the following quantities from smallest to largest: $f_x(3, 2), f_x(1, 2), f_y(3, 2), f_y(1, 2), 0$.



Problem 3. The figure below shows a contour plot of $f(x, y)$. Use the contour plot to determine the sign (positive, negative, or zero) of $f_x(P), f_y(P), f_{xx}(P), f_{yy}(P), f_{xy}(P)$.



Problem 4. Consider the following functions. Find their critical points and use the Second Derivative Test to classify them.

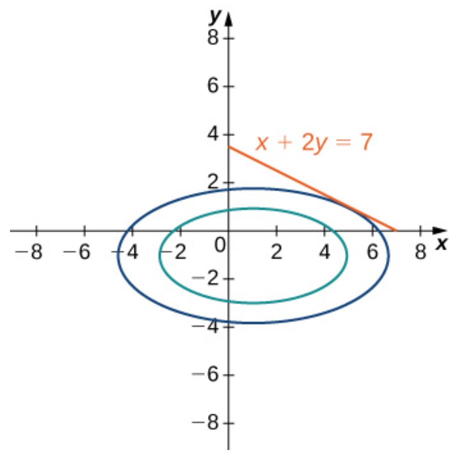
a. $f(x, y) = y^2 + xy + 3y + 2x + 3$

b. $f(x, y) = x^2 + 2y^2 - x^2y$

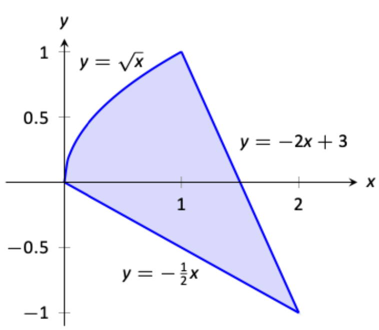
Problem 5. Find the global minimum and maximum value, as well as the location of where they occur, of the function $f(x, y) = 3y - 2x$ constrained to the region bounded by $y = x^2 - 2$ and $y = x$.

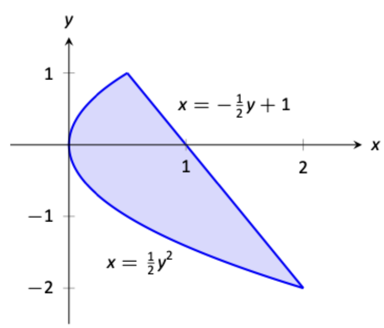
Problem 6. A food manufacturing company is planning to make cans in the shape of circular cylinders with a volume of 10 cubic centimeters. In order to save on the cost of labels (on sides as well as tops and bottoms), they would like to minimize the surface area of the can. Use the method of Lagrange multipliers to find dimensions (radius x and height y) that fit these requirements.

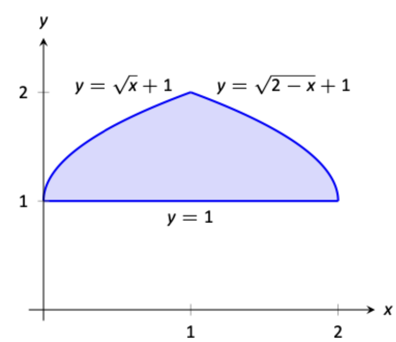
Problem 7. Let $f(x, y) = x^2 + 4y^2 - 2x + 8y$. Use the method of Lagrange Multipliers to determine the extreme values of f under the constraint that $x + 2y = 7$ and $x, y \geq 0$. In the figure below, label the z value of the level curve tangent to the line $x + 2y = 7$ and sketch in a vector in the direction of the gradient of f at the point where that level curve and line meet.



Problem 8. Consider each planar region R described below. Set up double integrals $\iint_R 1\, dA$ for the area of R in two ways: using $dA = dx dy$ and $dA = dy dx$.







Problem 9. For each description in polar coordinates below, make a sketch of the given region.

a. $r \geq 3$

b. $-\pi/2 \leq \theta \leq \pi/4$

c. $1 \leq r \leq 2, 3\pi/4 \leq \theta \leq 7\pi/4$

Problem 10. For each region in the xy-plane described below, express it using inequalities involving the polar variables r and θ.

a. The bottom half of the unit disk (centered at the origin)

b. The right half of the annulus with inner and outer radius 4 and 9 (centered at the origin)

c. The quarter of the unit disk in the third and fourth quadrants between the lines $y = \pm x$.