Math 203 — Spherical coordinates

Problem 1. For each point (ρ, θ, ϕ) given in spherical coordinates below, identify the sign of each component of its Cartesian coordinates (x, y, z). For example, if the point has positive x, negative y, and $z = 0$ your answer should be $(+, -, 0)$.

a. $(1, \pi/4, \pi/4)$

b. $(2, \pi, 3\pi/4)$

c. $(3, 5\pi/4, \pi/2)$

d. $(4, 7\pi/4, 5\pi/6)$

e. $(5, \pi/4, \pi)$

Problem 2. Describe the following regions using inequalities involving spherical variables ρ, θ, ϕ.

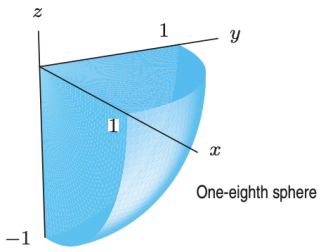
a. The quarter ball of radius 1, centered at the origin where $y \leq 0$ and $z \leq 0$.

b. The top half the solid region between spheres of radius 1 and 2 centered at the origin.

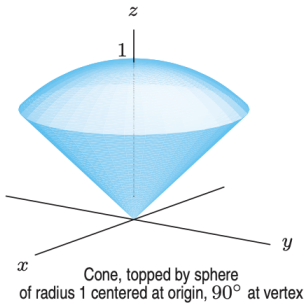
c. The plane $z = 1$.

d. The solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the plane $z = 1$.

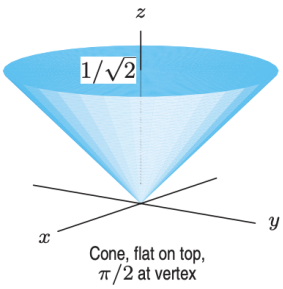
Problem 3. Set up a triple integral $\iiint_D (x + y + z)\, dV$ in spherical coordinates for each region below. Note the caption for the first region should be "one-eighth ball" since the region is a solid.



One-eighth sphere



Cone, topped by sphere
of radius 1 centered at origin, 90° at vertex



Cone, flat on top,
$\pi/2$ at vertex