Math 203 — Curl and Green's Theorem

Problem 1. Verify Green's Theorem by computing $\oint_C \mathbf{F} \cdot d\mathbf{r}$ and $\iint_R \operatorname{curl} \mathbf{F}\, dA$ for each of the following vector fields $\mathbf{F}$ and closed, positively oriented curves C which enclose regions R.

a. $\mathbf{F}(x,y) = \langle x+y, y\rangle$ and C is the unit circle.

b. $\mathbf{F}(x,y) = \langle 3y, 2x\rangle$ and C is the parabola $x = y^2$ connecting $(1,1)$ and $(1,-1)$ along with the line segment connecting these points.

Problem 2. Let $\mathbf{F}(x,y) = \langle y, x\rangle$ and $\mathbf{G}(x,y) = \langle 3y, -3x\rangle$. Let $C = C_2 - C_1$ as shown in the figure below, where C_1 is the line segment from $(-1,1)$ to $(1,-1)$ and C_2 is the portion of the positively oriented unit circle between these points.

a. Compute the curl of $\mathbf{F}$ and $\mathbf{G}$ and find the potential function of each vector field if possible.

b. State whether the Fundamental Theorem of Line Integrals or Green's Theorem can be used to compute each of the following line integrals. Compute the value of each using any valid method.

1. $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$
2. $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$
3. $\oint_C \mathbf{F} \cdot d\mathbf{r}$
4. $\int_{C_1} \mathbf{G} \cdot d\mathbf{r}$
5. $\int_{C_2} \mathbf{G} \cdot d\mathbf{r}$
6. $\oint_C \mathbf{G} \cdot d\mathbf{r}$