Math 206 — Exam 1 review guide

Your exam in class on October 11 will contain about 5 problems, some with multiple parts. You should expect to see a question asking you to state some definitions or theorems, a question asking you to prove a statement that was proved in class, and a few questions in the style similar to worksheets, homework, and quizzes. You should expect to write 2-3 proofs. It will cover material from Homework 0 to Homework 4. In the textbook, this is material spanning Chapters 2-8, along with some preliminary material on open set. There will be no material related to closed sets on this exam. I have outlined some important definitions, theorems, and genearl topics below. Also, the problems below give you a sampling of some problems like those that will appear on the exam, but it's not necessarily comprehensive, so make sure to review old homework, quizzes, worksheets, and lecture notes.

Definitions and theorems

While not necessarily comprehensive, here is an absolutely-must-know list of definitions.

- the empty set $\emptyset$, set of integers, set of natural numbers, set of rational numbers, even integer, odd integer, a divides b, absolute value of x
- for given sets A and B, $A \subseteq B, A \subsetneq B, A \not\subseteq B, A = B, A \cup B, A \cap B, A \setminus B, B \setminus A, A^c$
- for a given collection $\mathcal{A}$ of sets, $\bigcup_{A \in \mathcal{A}} A$ and $\bigcap_{A \in \mathcal{A}} A$
- open interval, closed interval, open set in $\mathbb{R}$

Similarly, here is an absolutely-must-know list of statements you should know.

- DeMorgan's Laws for logical statements and for sets
- the triangle inequality

Proofs and logic

You should know the following about proofs and logic. You should know how to:

- Fill out a truth table and decide whether statements are logically equivalent.
- Negate statements, including statements that involve "and," "or," quantifiers, inequalities, equalities, and implications.
- Write and structure direct proofs of implications (ie. if-then statements) and bi-implications (ie. if-and-only-if statements), contrapositive proofs, proofs by contradiction, proofs involving cases, proofs of for-all statements, and proofs of there-exists statements.
- Write proofs involving set containment, equality, and set operations.
- Write proofs using the triangle inequality.
- Prove a subset of $\mathbb{R}$ is open.

Sample problems

These problems are not comprehensive and there are more than you will see on the exam itself, but will give you an idea of the kinds of questions to expect.

Problem 1. Negate the following statements:

a. For all $x > 0$ there exists $y > 0$ such that $xy = 1$.

b. For every prime number p there is another prime number q such that $q > p$.

c. We have $0 < x < 10$ and it is also the case that $x \leq 2$ or $x \geq 3$.

Problem 2. Write the converse and the contrapositive of the following statements:

a. If $x > -3$ then $|(x+3)/(x+5)| < 1$.

b. If $x \in A \cup B$ then $x \in A$ or $x \in B$.

Problem 3. State whether each of the following is true or false.

a. For all $x \in \mathbb{Q}$ there exists $y \in \mathbb{Q}$ such that $x = 2y$.

b. There exists $y \in \mathbb{Q}$ such that for all $x \in \mathbb{Q}$, $x = 2y$.

c. For all $x \in \mathbb{Q}$ there exists $y \in \mathbb{Q}$ such that $x = y^2$.

d. The set $\bigcap_{n=1}^{\infty}(-1/n, 1+1/n)$ is open.

e. The set $\bigcup_{n=1}^{\infty}(-1/n, 1+1/n)$ is open.

Problem 4. Prove the following statements:

a. Let $a, b, c \in \mathbb{Z}$. If $a^2 \mid b$ and $b^3 \mid c$, then $a^6 \mid c$.

b. Let $a, b \in \mathbb{Z}$. If a and b have the same parity (ie. they're both even or both odd), then $3a + 7$ and $7b - 4$ do not have the same parity.

c. Let $\delta \in (0, 1)$ and suppose that $|x - 3| < \delta$. Prove that there exists a constant C, which does not depend on x nor on δ, such that $|2x^2 - 18| \leq C\delta$.

d. Let A, B, C be sets. If $A \subseteq B$ then $A \setminus C \subseteq B \setminus C$.