

Math 206 — Congruence modulo n

Problem 1. Consider the equation $8 \equiv x \pmod{6}$. Find 5 solutions of this equation. That is, find 5 values of $x \in \mathbb{Z}$ that satisfy the equation.

Problem 2. Find $[8]_6$.

Problem 3. Repeat the previous two problems with 0, 1, and 2 in place of 8.

Problem 4. The integers modulo 6, denoted \mathbb{Z}_6 , is a collection of equivalence classes like $[0]_6, [1]_6, [2]_6$, and so on. In other words, $\mathbb{Z}_6 = \{[m]_6 : m \in \mathbb{Z}\}$.

- Explain why it is the case that for every $m \in \mathbb{Z}$ there exists $r \in \mathbb{Z}$ such that $0 \leq r < 6$ and $m \equiv r \pmod{6}$.
- Explain why it is the case that for every $m \in \mathbb{Z}$ there exists $r \in \mathbb{Z}$ such that $0 \leq r < 6$ and $[m]_6 = [r]_6$.
- Explain why $\mathbb{Z}_6 = \{[0]_6, [1]_6, [2]_6, [3]_6, [4]_6, [5]_6\}$.

Problem 5. Prove the division algorithm in the case when $n < 0$.