Math 241, Spring 2022 — Homework 2

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Due February 9 at 5:00 pm

Instructions. This problem set covers material from Week 2 of class, along with a reading about chaos. The exercises below come from Chapter 4 on page 42.

Problem 1. Chaos theory is a subfield within dynamical systems that is aimed at studying systems that behave erratically in some sense. They are truly deterministic systems like the ones we have seen in our class so far (the behavior of the orbit of an initial seed is determined by iterating a map repeatedly), but it's difficult to summarize their behavior like you'll do in the rest of this assignment. Read the article The Hidden Heroines of Chaos (or listen to it; there is a podcast version of the article playable at the top of the page), talk to a friend or family member about it, and respond to the following prompts.

1. What stood out to you about the article? What were some key ideas or pieces of the story?
2. What came up in your conversation with your friend or family member?
3. Anything else you want to mention or ask?

Problem 2. Do the following parts of Exercises 1 and 3. For each part, you should draw a cobweb diagram by hand (you can use MATLAB to help you, but I'd like you to submit a hand drawing), draw a phase portrait, and then give a case by case summary of how the orbit of x_0 behaves for all $x_0 \in \mathbb{R}$. That is, you should say which points are fixed points or periodic points or eventually fixed/periodic, and then describe the long term behavior of initial seeds that are not fixed, periodic, or eventually fixed/periodic. You can use bullet points or a paragraph to write out this summary.

1. Part c
2. Part f (note that both 0 and 1 are special in some way)
3. Part g

Problem 3. Do the following parts of Exercise 4, following the same structure as in Problem 1 (draw cobweb diagrams, phase portraits, and write a case by case summary of orbit behaviors).

1. Part a
2. Part c (you should find that $-1, 0$, and 1 are each special in some way)
3. Part e (make sure to talk about periodic points as well as fixed points)