Math 241, Spring 2022 — Homework 3

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Due February 16 at 5:00 pm

Instructions. This problem set covers material from Week 3 of class. The exercises below come from Chapter 5 on page 59.

Problem 1. Do the following exercises from Chapter 5.

1. Exercise 1, parts c, d, f, h

2. Exercise 4, parts a, c

Problem 2. For each of the following functions, (1) find its fixed points, (2) find the value of $F'(x)$ at each of the fixed points and characterize them as attracting, repelling, or neutral, and (3) draw a phase diagram to summarize the behavior of orbits. You might use MATLAB or a hand drawn cobweb diagram to check your ideas, but you only need to submit the 3 things above.

1. $F(x) = 2x - x^2$

2. $F(x) = x^2 - x$

Problem 3. Let $F(x) = ax - x^2$ where $a \in \mathbb{R}$ is a given constant.

1. Find the fixed points of F. Note that 0 should be one of your fixed points and the other will be in terms of a.

2. Find the value of $F'(x)$ at each of the fixed points.

3. Find all values of a that make 0 a repelling fixed point and the other an attracting fixed point.

Problem 4. Using the proof of the Attracting Fixed Point Theorem we did in class on Thursday as a template, write a proof to explain why the following theorem (called the Repelling Fixed Point Theorem) works.

Theorem. *Suppose p is a repelling fixed point for F (meaning $|F'(p)| > 1$). Then there is an interval I that contains p in its interior and in which the following condition is satisfied: if $x_0 \in I$ and $x_0 \neq p$, then there is an integer $n > 0$ such that $F^n(x_0) \notin I$.*