REMEMBER TO RECORD MEETING!

- Tim / Prof. / Prof. Chumley (he/him)
- Moodle - announcements
- Webpage (tchumley.mtholyoke.edu/m241)
  - notes, schedule, assignments, syllabus
- Gradescope
- Piazza
- Homework - weekly, due Wednesdays at 5 pm in general
  - redos, due Fridays at 5 pm in general
- Two exams
- End of semester project (presentation, report)
- Participation (class discussions, Piazza, surveys)
- Office hours (tentative)
  - Mondays, Wednesdays, 1-2 pm
  - Tuesdays, Thursdays, 2-3 pm

  no appointment necessary, drop in

  - on Zoom for first two weeks

Introduction (focused on Section 2.2)

What is a dynamical system?

Informal definition: it's a mathematical model of a real world system that evolves over time.

Definition A deterministic discrete-time dynamical system is a set X called the state space together with a function $F : X \to X$ that tells us how the system evolves.

We start with an initial seed $x_0 \in X$

and repeatedly apply F

$$x_1 = F(x_0)$$

$$x_2 = F(x_1) = F(F(x_1))$$

$$x_3 = F(x_2) = F(F(F(x_0)))$$

The infinite list of states

$$x_0, x_1, x_2, x_3, \ldots$$

is called the orbit of x_0 and

x_n is called the n^{th} iterate of x_0.

$$x_n = F(F(F(\ldots(x_0))))$$

$$= F \circ F \circ \cdots \circ F(x_0)$$

$$= F^n(x_0). \quad \text{(n-times composition)}$$

Example Suppose we want to model the growth or decay of a population of a certain species over time (eg. generation to generation). The species grows/decays by a factor/multiple r of the current generation. Let $P_0, P_1, P_2, \dots$ be size of the population in generation $0, 1, 2, \dots$

① What are F and X?

$$X = [0, \infty)$$

$$P_{n+1} = r P_n$$

$$F(x) = rx$$

$$P_1 = F(P_0) = r P_0$$

$$P_2 = F(P_1) = r(r P_0) = r^2 P_0$$

$$P_3 = F(P_2) = r P_2 = r(r^2 P_0) = r^3 P_0$$

$$\vdots$$

$$P_n = r^n P_0$$

(2) If $r = 0.5$, what happens for

$n = 10, 100, 10000, \ldots$

$$P_n = (.5)^n P_0 \to 0 \text{ as } n \to \infty.$$

(3) If $r = 2$, what happens

for large n?

$$P_n = 2^n P_0 \to \infty \text{ as long as } P_0 > 0.$$

Problem 1. Suppose a population of a certain species has a maximum possible size, and if the population ever reaches that maximum size, it will die out in the next generation. Let P_n denote the fraction of the maximum size that's alive in generation n. Suppose that

$$P_{n+1} = 2P_n(1 - P_n).$$

This population model is called the *logistic model* and the value 2 is a parameter of the system that I've chosen somewhat arbitrarily for the purposes of the introductory questions below.

1. What are the *state space* X and *map* F of this system?
2. Suppose the population size P_n is small in generation n. Roughly speaking, what happens to the population size P_{n+1} in generation $n+1$?
3. Suppose the population size P_n is big in generation n. Roughly speaking, what happens to the population size P_{n+1} in generation $n+1$?
4. Using a calculator, what happens to the population after 10 generations when $P_0 = 0.2$? When $P_0 = 0.7$? List the *orbit* of each of these initial values.
5. What do you think the term *fixed point* means?
6. Do you think the system has any fixed points? How many? What do you think they are?
7. What equation does a fixed point have to satisfy? Try using algebra to show that the system has 2 fixed points and to find what they are.

1 $\quad X = [0, 1]$

$$F(x) = \underline{2x(1-x)}$$

2

$$P_{n+1} = 2P_n(\underbrace{1 - P_n}) $$

P_n small $\Rightarrow 1 - P_n$ close to 1

$\Rightarrow P_{n+1} \approx 2P_n$

(3) P_n close to 1 $\Rightarrow$ $1-P_n$ close to 0

P_{n+1} goes close to 0.

(4) We saw each orbit approach 0.5 using the time_series.m MATLAB script.

(7) x is a fixed point if it satisfies the equation

$$x = F(x)$$

(applying F doesn't change the state)

After class: Work on Homework 0.

We'll use MATLAB next time, so try installing it!

- Can you stay for a few minutes if you're on the waitlist?