Plan for Tuesday

- Discuss period doubling bifurcation for

  $Q_c(x) = x^2 + c$ (worksheet problems)

- Worksheet on tangent, period doubling in context of logistic map

- Bifurcation diagrams?

$$F_\lambda(x) = \lambda x(1-x)$$

Fixed points

$$x = \lambda x(1-x)$$

$$= \lambda x - \lambda x^2$$

$$0 = (\lambda-1)x - \lambda x^2$$

$$= x(\lambda - 1 - \lambda x)$$

$$x = 0, \quad x = \frac{\lambda-1}{\lambda}$$

Only one fixed point when

$\lambda = 0$, $\lambda = 1$

Two fixed points when

$\lambda \neq 0, 1$

Attraction / repulsion

$$F_\lambda'(x) = \lambda - 2\lambda x$$

$F_\lambda'(0) = \lambda$, so 0 is attracting when $|\lambda| < 1$

0 is neutral when $|\lambda| = 1$

0 is repelling when $|\lambda| > 1$

$$F_\lambda'\left(\frac{\lambda-1}{\lambda}\right) = \lambda - 2\lambda\left(\frac{\lambda-1}{\lambda}\right)$$

$$= \lambda - 2(\lambda - 1)$$

$$= \lambda - 2\lambda + 2$$

$$= 2 - \lambda$$

$|2-\lambda| < 1$ when

$-1 < 2 - \lambda < 1$

$-3 < -\lambda < -1$

$3 > \lambda > 1$

$\frac{\lambda-1}{\lambda}$ is attracting when $1 < \lambda < 3$

$\frac{\lambda-1}{\lambda}$ is neutral when $\lambda = 1, 3$

$\frac{\lambda-1}{\lambda}$ is repelling when $\lambda < 1$ or $\lambda > 3$

Period-doubling

Investigate the graph of F_λ^2 using Desmos. Where does a period-doubling bifurcation occur? What seems to be the relationship between period doubling and neutral fixed points?

Look at $\lambda = -1$, $\lambda = 3$

(this is where $F_\lambda(x)$ has -1 slope at fixed points)

It seems neutral fixed point with -1 slope yields period doubling bifurcation