Plan for Thursday

- Discussion of Tuesday worksheet

- Official definitions of saddle node and period doubling bifurcations

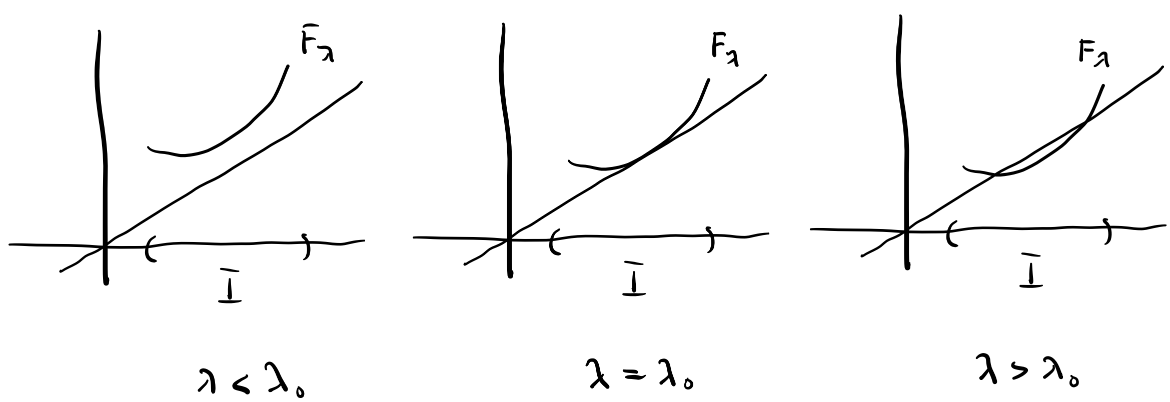
Now is a good moment to finally spell out the definitions of saddle-node and period-doubling bifurcations.

Def A family of functions F_λ undergoes a <u>saddle-node bifurcation</u> at λ_0 if there exists an open interval I and a value $\varepsilon > 0$ so that

1. when $\lambda \in (\lambda_0 - \varepsilon, \lambda_0)$, F_λ has no fixed points in the interval I

2. when $\lambda = \lambda_0$, F_λ has one fixed point in I

3. when $\lambda \in (\lambda_0, \lambda_0 + \varepsilon)$, F_λ has two fixed points in I



<u>Def</u> A family of functions F_λ undergoes a <u>period-doubling bifurcation</u> at λ_0 if there exists an open interval I and a value $\varepsilon > 0$ so that

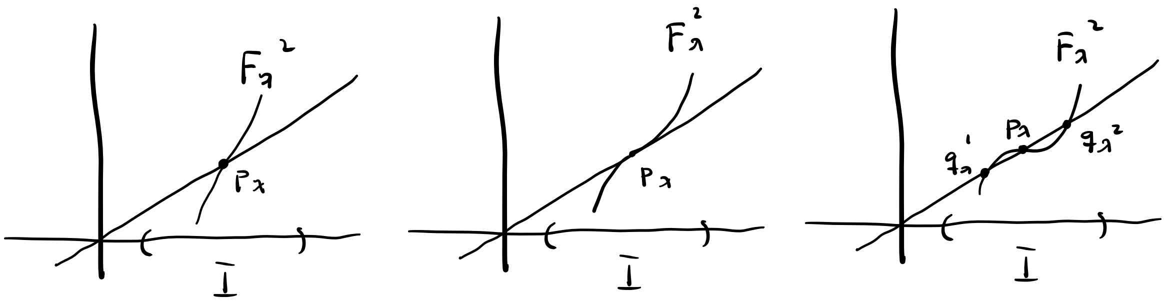
① for all $\lambda \in (\lambda_0 - \varepsilon, \lambda_0 + \varepsilon)$, F_λ has a fixed point $p_\lambda \in I$

② when $\lambda \in (\lambda_0 - \varepsilon, \lambda_0)$, p_λ is attracting/repelling and F_λ has no 2-cycles in I

③ when $\lambda = \lambda_0$, p_λ is neutral and F_λ still has no 2-cycles in I

④ when $\lambda \in (\lambda_0, \lambda_0 + \varepsilon)$, p_λ is repelling/attracting and F_λ has a 2-cycle formed by $q_\lambda^1, q_\lambda^2 \in I$.

⑤ As $\lambda \to \lambda_0$, $q_\lambda^1, q_\lambda^2 \to p_{\lambda_0}$



$\lambda < \lambda_0$ $\qquad\qquad$ $\lambda = \lambda_0$ $\qquad\qquad$ $\lambda > \lambda_0$

Remark

When p is a neutral fixed point of F with $F'(p) = -1$,

p is also a fixed point for F^2

since $F^2(p) = F(F(p)) = F(p) = p$.

and $(F^2)'(p) = F'(p) \cdot F'(p) = (-1)(-1) = 1$

so F^2 is tangent to $y = x$, which is exactly what happens when a saddle node bifurcation occurs.