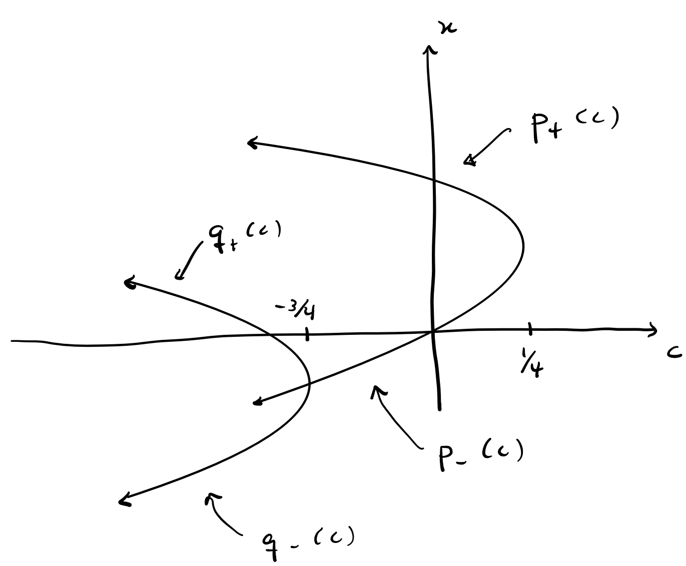
Bifurcation diagrams

We can plot fixed points and periodic points on the $\lambda - x$ plane (or $c - x$) to visualize bifurcations.

Example $Q_c(x) = x^2 + c$ has

- fixed points $p_\pm = \dfrac{1 \pm \sqrt{1-4c}}{2}$ when $c < \frac{1}{4}$
- period 2 points

$$q_\pm = \frac{-1 \pm \sqrt{1-4(c+1)}}{2} \quad \text{when } c < -\frac{3}{4}$$



Remark We'd like to keep doing this with higher order periods, but it can be difficult to find periodic points analytically.

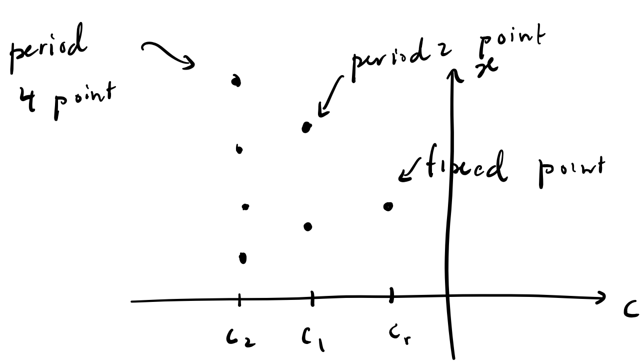
Experiment Let's plot the asymptotic behavior of O for a collection of c values using numerical simulation.

For each c value below, we'll plot points in orbit O.

If O converges to a fixed point p, we'll plot (c, p).

If O converges to a 2-cycle, q_1, q_2, we'll plot (c, q_1) and (c, q_2).

If O doesn't seem to stabilize, we'll plot the last 10 points in the orbit



c-values to use (equally spaced in each interval)

- 5 c's in interval $(-0.75, 0.25)$
- 5 in $(-1.25, -0.75)$
- 20 in $(-1.4, -1.25)$
- 5 in $(-1.75, -1.4)$
- 10 in $(-1.78, -1.76)$
- 5 in $(-2, -1.78)$

Questions
- How many iterations should be done?
- What tools should be used to do the iterations?
- How many decimal places to record?
- How should results be stored?