Another method for ternary expansions

Example

$$\frac{5}{8} = \frac{s_1}{3} + \frac{s_2}{3^2} + \frac{s_3}{3^3} + \cdots$$

Multiply by 3 on both sides

$$\underbrace{\frac{15}{8}}_{1+\frac{7}{8}} = \underbrace{s_1}_{=0,1,\text{or } 2} + \underbrace{\frac{s_2}{3} + \frac{s_3}{3^2} + \cdots}_{\leq 1}$$

To make both sides equal, we need

s_1 to be 1

and we need

$$\frac{7}{8} = \frac{s_2}{3} + \frac{s_3}{3^2} + \frac{s_4}{3^3} + \cdots$$

Multiply both sides by 3:

$$\underbrace{\frac{21}{8}}_{2+\frac{5}{8}} = \underbrace{s_2}_{=0,1,\text{or } 2} + \underbrace{\frac{s_3}{3} + \frac{s_4}{3^2} + \cdots}_{\leq 1}$$

We need $s_2 = 2$ and

$$\frac{5}{8} = \frac{s_3}{3} + \frac{s_4}{3^2} + \cdots$$

Now the process seems to repeat!

We'll get $s_3 = 1$, $s_4 = 2$, ...

So $\frac{5}{8} = 0.\overline{12}$