# Chapter 4    Graphical Analysis
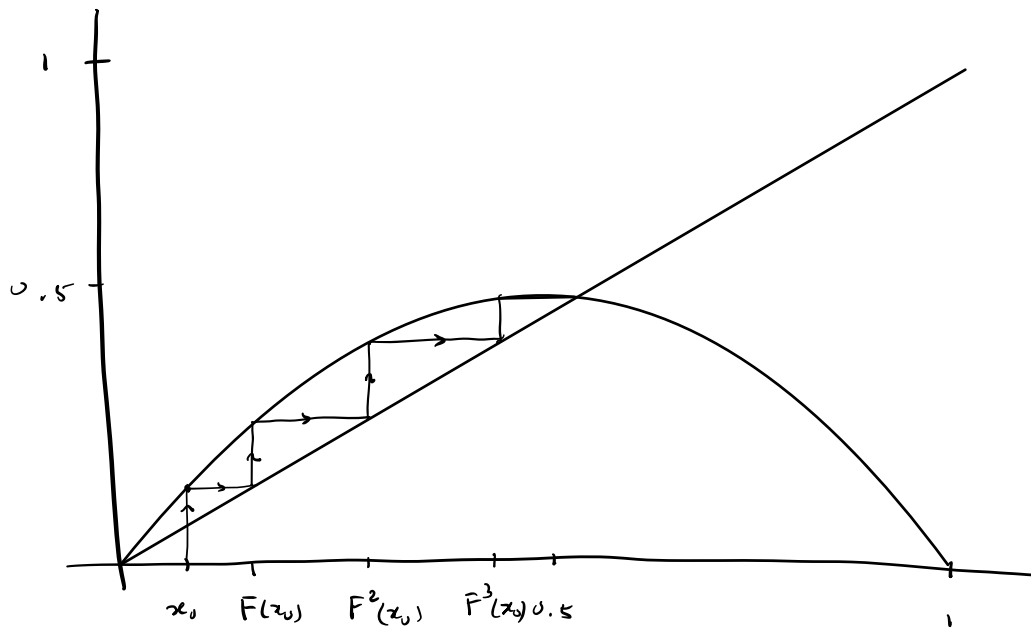
Today we're going to learn a new technique for visualizing the orbit of a point $x_0$ that will help us understand long term behavior (eg. attracting/repelling fixed points). The technique is called a cobweb diagram.

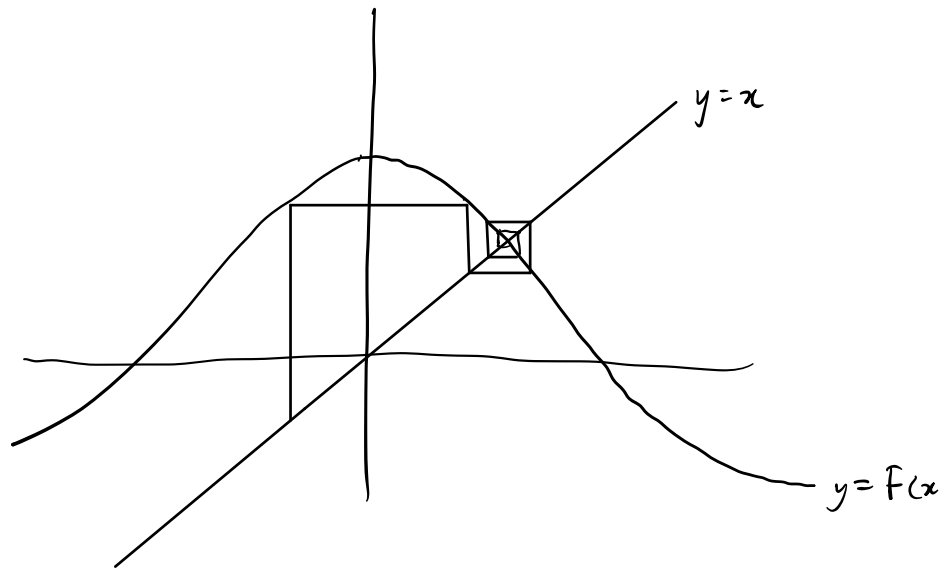Example    Let $F(x) = 2x(1-x)$ and plot the orbit of $x_0 = 0.1$ in a cobweb diagram. What happens to any orbit that starts with $x_0 > 0.5$?
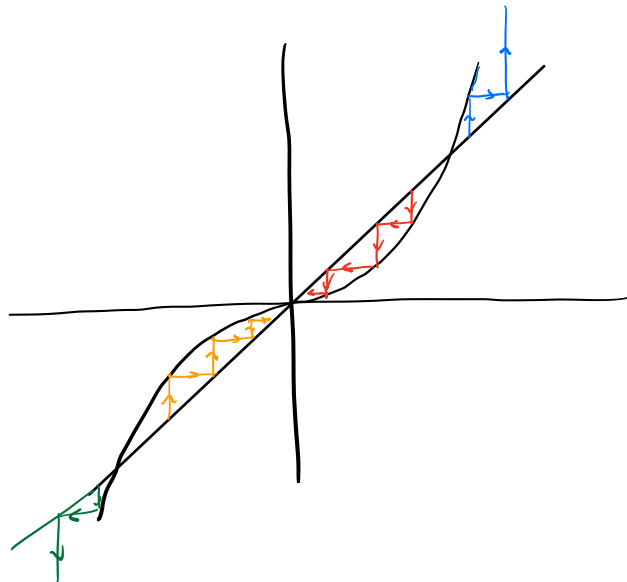
Graph axis labels: $x_0$, $F(x_0)$, $F^2(x_0)$, $F^3(x_0)$, $0.5$, along the horizontal axis; $0.5$ and $1$ on the vertical axis; $1$ on the horizontal axis at right.

The procedure:

① Start at the point $(x_0, x_0)$ and draw vertical line to $(x_0, F(x_0))$

② draw horizontal line to $(F(x_0), F(x_0))$

③ repeat process by drawing vertical line from $(F(x_0), F(x_0))$ to $\left( F(x_0), F^2(x_0) \right)$

and so on.

**Example** Consider $F(x) = \cos x$. Use a cobweb diagram to demonstrate the fact that orbits tend a fixed point

$y = x$

$y = F(x$

Example   Give   a   breakdown   of   all   the

possible   behaviours   of   orbits   for   $f(x) = x^3$



— when   $|x_0| < 1$,   the orbit   of $x_0$

converges   to   $O$

— when   $|x_0| > 1$,   the orbit   of $x_0$

diverges   to   $\pm \infty$